MEC2120
Kinematics of Machines

Syllabus

Unit – I : Kinematic pairs & chain, constrained criterion, mobility and range of movement, Planar Mechanisms and its inversion, Straight line motion Mechanisms, Pantograph, Engine indicator, Hook's joint and steering gear mechanism.

Unit–II : Velocity analysis in mechanism: relative velocity & Instantaneous centre method, Acceleration analysis in mechanism, Graphical method, problem involving Corriolis acceleration, Klien's construction, Analytical methods for velocity & acceleration analysis.

Unit–III : Kinematic Synthesis of Planar Mechanisms: Chebyshev Spacing of Precision Points, Two-/Three- position synthesis of Planar four bar mechanisms, Path Generation and Function generation problems, Bloch's Method and Freudenstein's method of synthesis.

Unit–IV : Gear Drives: Introduction, classification of gear, gear nomenclature, tooth profile, interference, path of contact, arc of contact of meshing gears. Gear Train: Simple, compound and epi-cyclic gear trains.

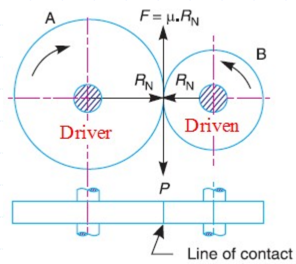
Books:

1. S S Ratan: Theory of Machines; McGraw Hill.
2. J S Rao: Mechanism & Machine Theory, New Age International.
3. Chales E Wilson & J Peter Sadler: Kinematics & Dynamics of Machinery; Pearson Education.

Unit 4

Gears and Gear Drives

Friction wheel



- *For no Slipping condition, the following kinematic relationship exists.*

$$v_p = \omega_A r_A = \omega_B r_B$$
$$\Rightarrow \quad \frac{\omega_A}{\omega_B} = \frac{r_B}{r_A}$$
$$\frac{N_A}{N_B} = \frac{r_B}{r_A}$$

- The friction drive is not a positive drive.
- The friction wheels can only be used for transmission of small powers.

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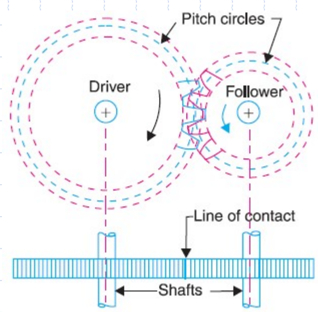
Toothed wheel/Gear



- A friction wheel with the teeth cut on it is known as ***toothed wheel* or *gear*.**

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Toothed wheel/Gear

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

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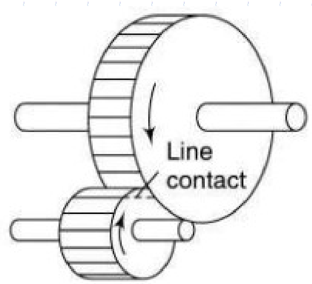
Classification of Toothed wheel/Gear

1. ***According to the position of axes of the shafts.***

   (*a*) Parallel, (*b*) Intersecting, and (*c*) Non-intersecting and non-parallel.

(*a*) Parallel,

spur gears



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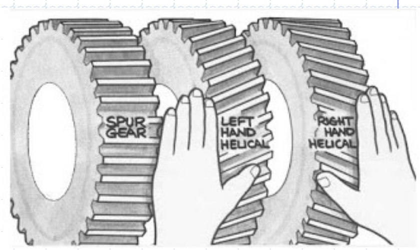
Classification of Toothed wheel/Gear

1. ***According to the position of axes of the shafts.***

   (*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Parallel,

helical gearing



- Mating helical gears (on parallel shafts) must have the same helix angle but the opposite hand.

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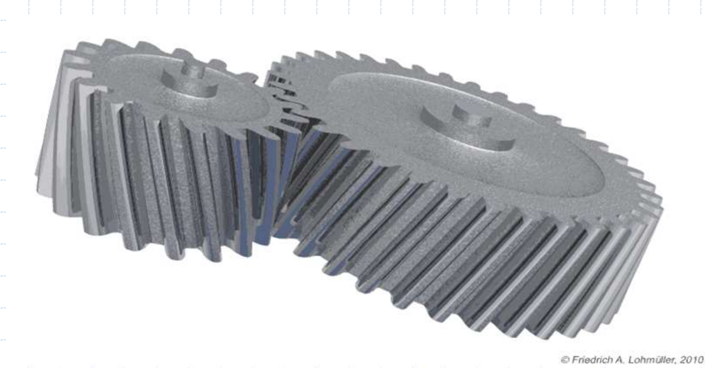
(*a*) Parallel,

helical gearing



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Classification of Toothed wheel/Gear

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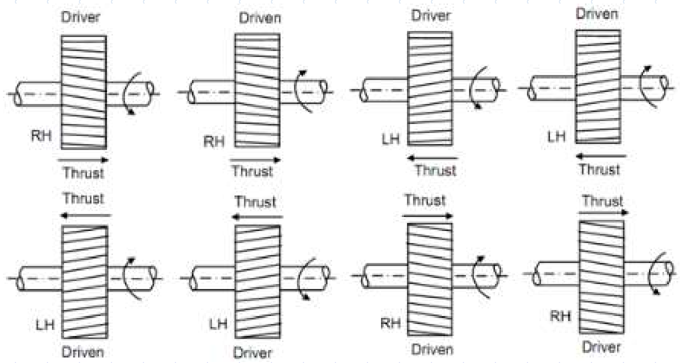


Classification of Toothed wheel/Gear

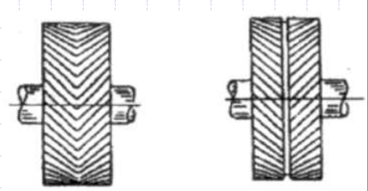
1. ***According to the position of axes of the shafts.***

   (*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Parallel,

herringbone gears

- To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces.
- Herringbone gears are mostly used on heavy machinery.



Herringbone Gear

Double-Crossed Helical Gear

Classification of Toothed wheel/Gear

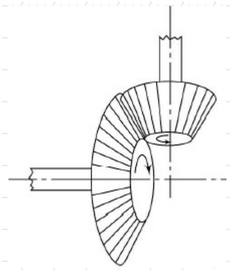
1. According to the position of axes of the shafts.

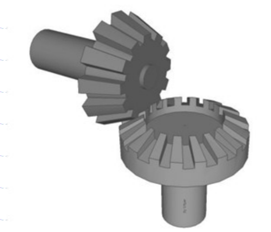
(***a***) Parallel, (***b***) Intersecting, and (***c***) Non-intersecting and non-parallel.

(*a*) Intersecting

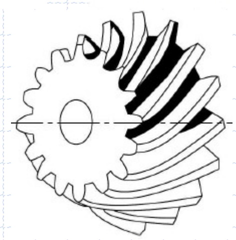
Bevel gears

Straight Bevel gears





Spiral or Helical Bevel gears



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Classification of Toothed wheel/Gear

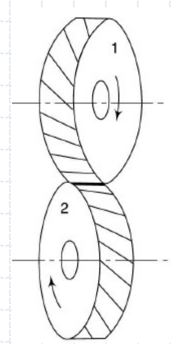
1. According to the position of axes of the shafts.

(***a***) Parallel, (***b***) Intersecting, and (***c***) Non-intersecting and non-parallel.

(*a*) Non-intersecting and non-parallel

Crossed Helical gears

- **These are used for light loads.**
- **By a suitable choice of helix angle for the mating gears, the two shafts can be set at any angle**
- **These gears theoretically give point contact, unlike spur or helical gears.**
- **Used for feed mechanism on machine tools, camshafts and oil pumps on small IC engines.**



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Classification of Toothed wheel/Gear

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   (*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Non-intersecting and non-parallel

Worm gears

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Classification of Toothed wheel/Gear

1. ***According to the position of axes of the shafts.***

   (*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Non-intersecting and non-parallel

Worm gears

- **Used to connect skew shafts (generally at right angles).**
- **Transmit high velocity ratios**
- **The worm gears give line contact between mating teeth unlike a point contact in the case of spiral gears (crossed helical gears).**
- **The load carrying capacity is improved by using enveloping worm, instead of a straight worm.**

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Classification of Toothed wheel/Gear

1. According to the position of axes of the shafts.

(*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Non-intersecting and non-parallel

Worm gear set types

1. Non-throated: The contact between the teeth is concentrated at a point.
2. Single-throated: Gear teeth are cuved to envelop the worm. There is a line contact between the teeth.
3. Double throated: There is area contact between the teeth.

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Classification of Toothed wheel/Gear

2. According to the peripheral velocity of the gears.

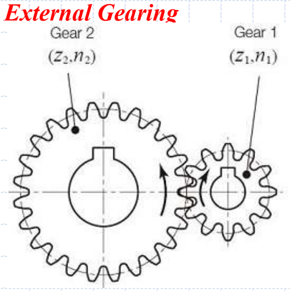
(*a*) Low velocity: Velocity is less than 3 m/sec,

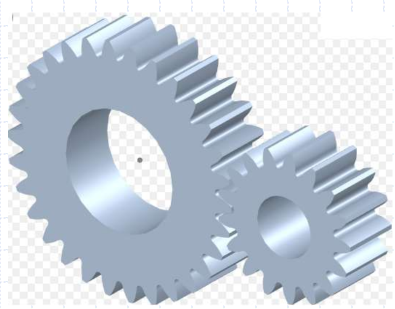
(*b*) Medium velocity: Velocity is between 3 m/sec to 15 m/sec, and

(*c*) High Velocity: Velocity is above 15 m/sec.

3. According to the type of gearing.

(a) External Gearing





- External gears have the gear teeth generated on the outside diameter of the component.
- In the case of external toothing, the teeth are directed outwards on the circumference.

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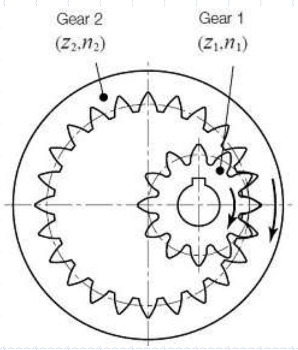
Classification of Toothed wheel/Gear

3. According to the type of gearing.

(b) Internal Gearing



- Internal Gears are **gear teeth generated in the internal diameter of a cylinder**.
- In the **case of internal gears, the teeth are directed inwards**.

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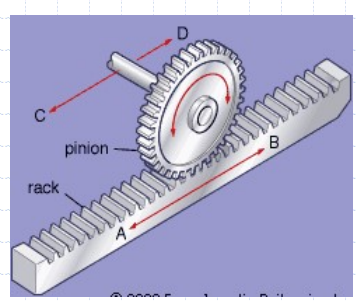
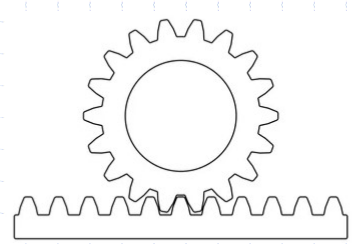
Classification of Toothed wheel/Gear

3. According to the type of gearing.

(c) Rack and Pinion

- A bar of rectangular cross section (the rack), having teeth on one side that mesh with teeth on a small gear (the pinion).
- If the pinion rotates about a fixed axis, the rack will translate; *i.e.,* move on a straight path. e.g. Some automobiles have rack-and-pinion drives on their steering mechanisms that operate in this way.
- If the rack is fixed and the pinion is carried in bearings on a table guided on tracks parallel to the rack, rotation of the pinion shaft will move the table parallel to the rack. e.g. On machine tools, rack-and-pinion mechanisms are used in this way to obtain rapid movements of worktables





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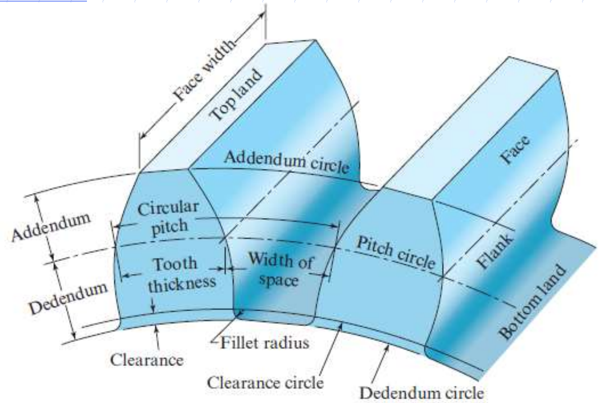
Gears



1. *Pitch circle.*
2. *Pitch circle diameter.*
3. *Pitch point.*
4. *Pitch surface.*
5. *Addendum circle.*
6. *Dedendum circle.*
7. *Addendum.*

8. Dedendum.
9. Total depth
10. Clearence
11. Clearence circle
12. Top land.
13. Bottom land
14. Face

15. Flank
16. Tooth profile
17. Face Width
18. Tooth thickness
19. Tooth space
20. Circular pitch, $P_c = \pi D/T$

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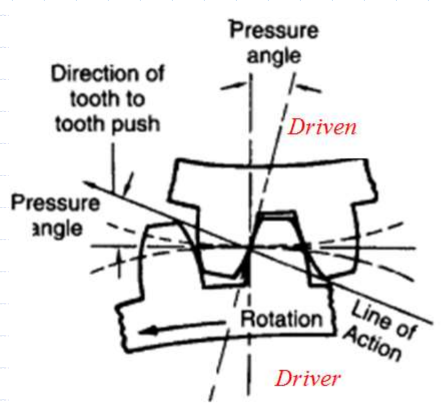
Gears

21. Diametral Pitch, $P_d = T/D = \pi/P_c$
22. Module, $m = D/T$

23. Line of Action
24.Pressure angle
25. Path of contact
26. Arc of contact



(Wheel 2)

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Law of Gearing

Q: Point of contact of two teeth.

T T : common tangent and

MN :common normal to the curves at the point of contact *Q*.

v_1 and v_2 : Velocities of the point *Q* on the wheels 1 and 2 respectively.

For the teeth to remain in contact, we must have

$$v_1 \cos\alpha = v_2 \cos\beta$$

$$(\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$$

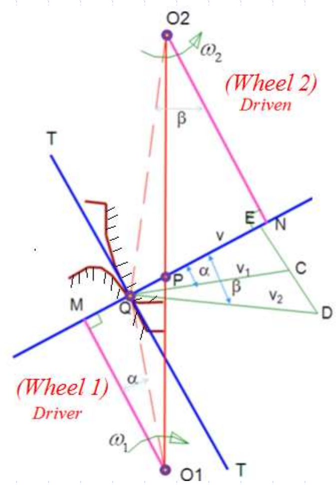
$$(\omega_1 \times O_1Q)\frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q)\frac{O_2N}{O_2Q}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \qquad Eq.(1)$$

Also we have

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad (\because \Delta O_1MP \sim \Delta O_2NP)$$

$Eq.(2)$



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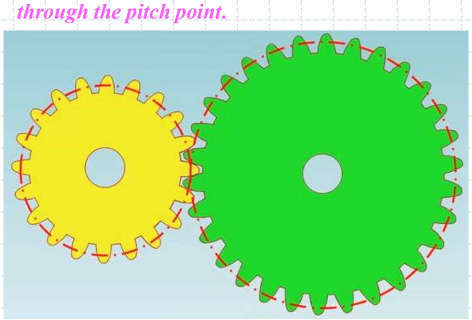
Law of Gearing

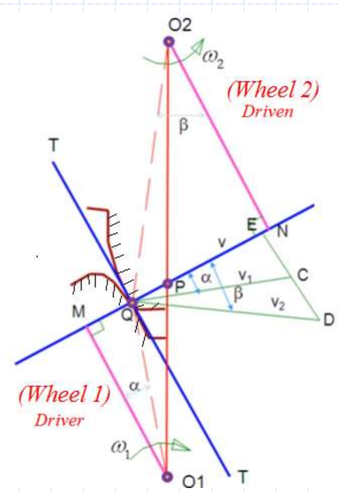
Combining Eq.1 and Eq.2

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

∴ For constant angular velocity ratio for all positions of the wheels, the point *P* must be the fixed point (called pitch point) for the two wheels.

Law of Gearing: ***The common normal at the point of contact between a pair of teeth must always pass through the pitch point.***





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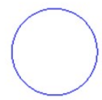
Teeth Profiles satisfying the law of gearing

1) *Involute Profile*
2) *Cycloidal Profile*



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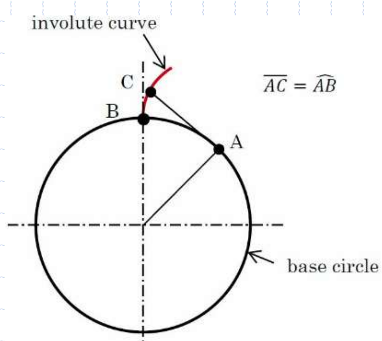
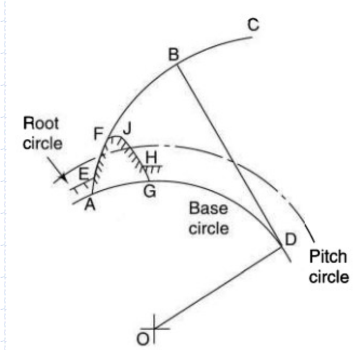
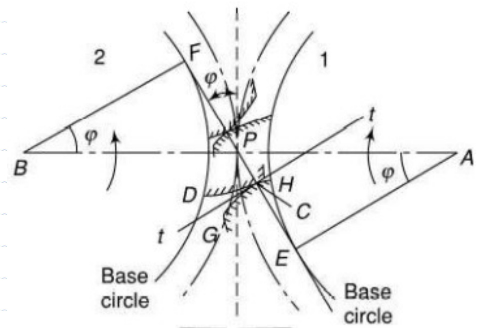




Teeth Profiles satisfying the law of gearing

1) Involute Profile

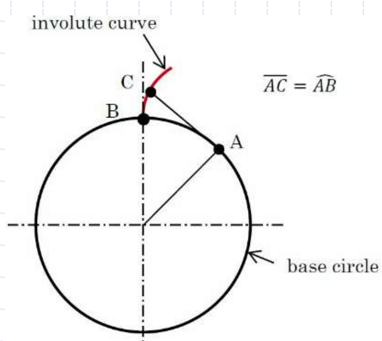


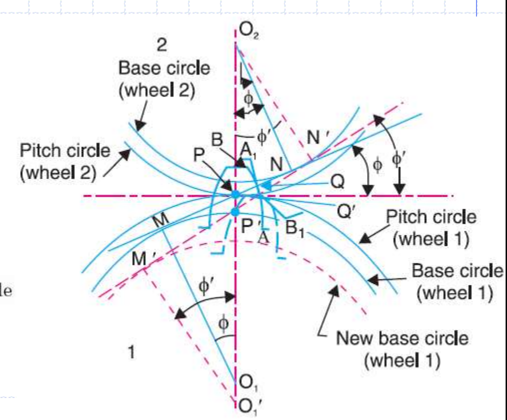
$$\text{velocity ratio of gears} = \frac{BP}{AP} = \frac{BF}{AE} = \text{constant}$$

Thus, for a pair of involute gears, the velocity ratio is inversely proportional to the pitch circle diameters as well as base circle diameters.

Teeth Profiles satisfying the law of gearing

1) Involute Profile

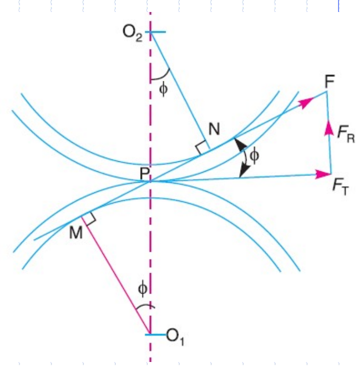




Teeth Profiles satisfying the law of gearing

Properties of Involute Profile

- The shape of involute profile is dependent only on the dimensions of the base circle.
- Angular velocity ratio of involute profile teeth is independent to centre distance of the base circles.
- if the centre distance is changed within limits, the velocity ratio remains unchanged. However, the pressure angle increases with the increase in the centre distance.
- When two involutes are in mesh, the angular velocity ratio is inversely proportional to the size of the base circles.
- The pressure angle of two involutes in mesh is constant.



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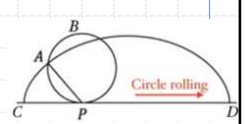
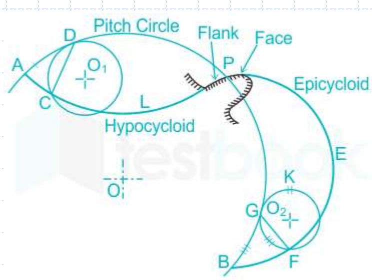
Teeth Profiles satisfying the law of gearing

Cycloidal Profile



The cycloid is defined as the locus of a point on the circumference of a circle that rolls without slipping on a fixed straight line.
In Cycloidal tooth profile, the faces of the teeth are epicycloids and the flanks are the hypocycloids.
An epicycloid is the locus of a point on the circumference of a circle that rolls without slipping on the circumference of another circle.
A hypocycloid is the locus of a point on the circumference of a circle that rolls without slipping inside the circumference of another circle.
Formation of cycloidal tooth :



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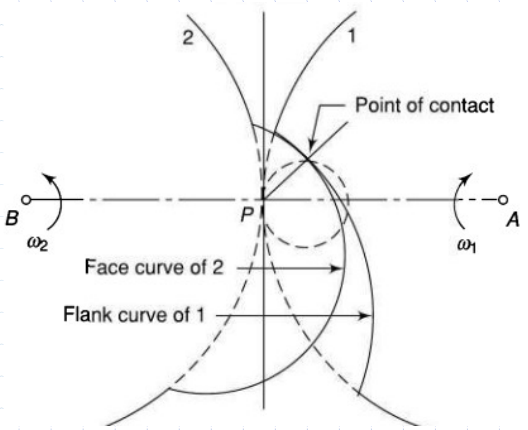
Teeth Profiles satisfying the law of gearing

Cycloidal Profile



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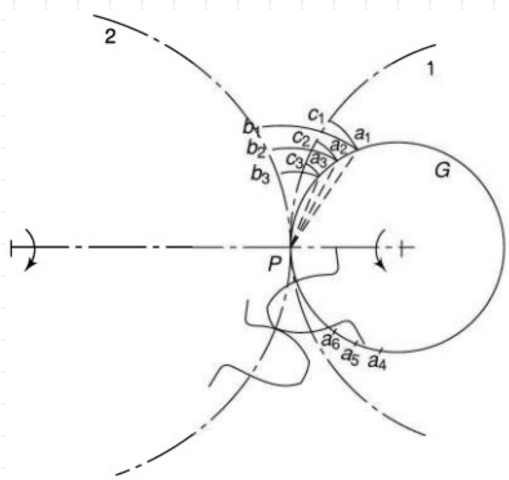
Teeth Profiles satisfying the law of gearing

Cycloidal Profile



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Teeth Profiles satisfying the law of gearing

Cycloidal Profile

Properties of Cycloidal Profile

- In case of cycloidal teeth, the pressure angle varies from the maximum at the beginning of the engagement to zero when the point of contact coincides with pitch point and then again increased to a maximum in the reverse direction. It results in changing bearing reactions at the support.
- These gears must be operated at exactly the correct centre distance.
- Since cycloidal teeth are made up of the two curves, it is very difficult to produce accurate profiles. This has rendered this system obsolete.

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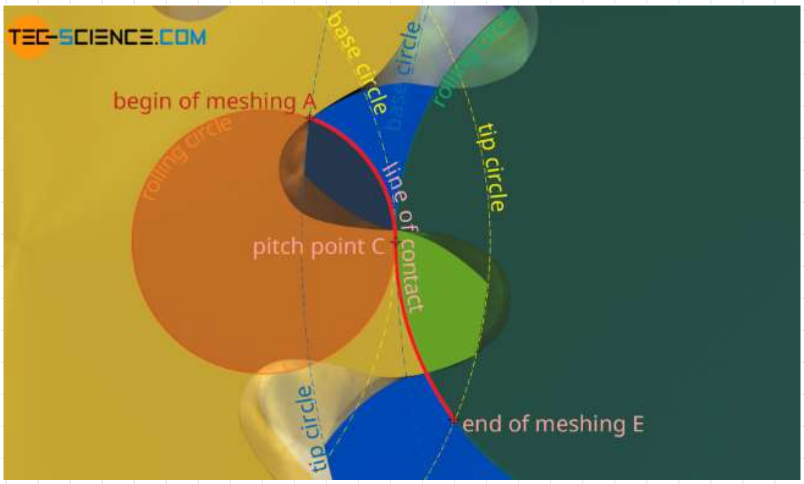
Meshing Cycloidal Gears



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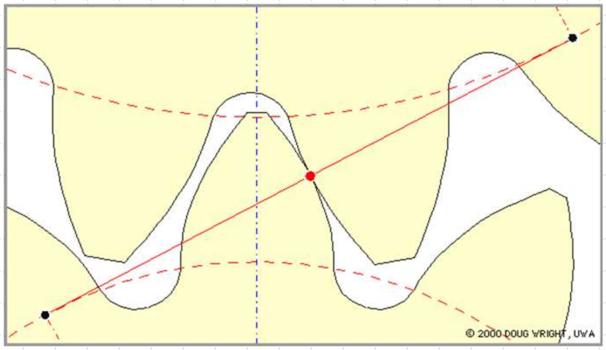
Path of Contact

Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement. Its length is called *length of path of contact*.



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Path of Contact

Length of Path of Approach

$$CP = CF - PF = \sqrt{R_a^2 - R^2 cos^2\emptyset} - R\sin\emptyset$$

Length of Path of Recess

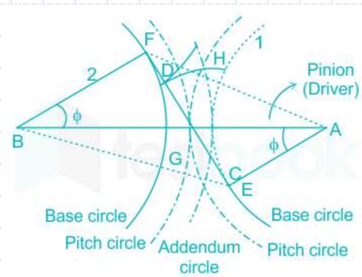
$$PD = DE - PE = \sqrt{r_a^2 - r^2 cos^2\emptyset} - r\sin\emptyset$$

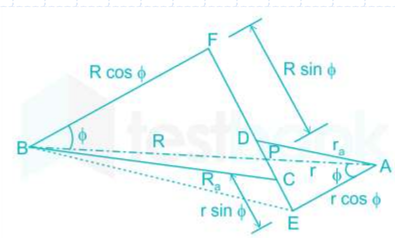
Length of Path of Contact

$$= CD = CP + PD$$

$$= \sqrt{R_a^2 - R^2 cos^2\emptyset} + \sqrt{r_a^2 - r^2 cos^2\emptyset} - (R + r)\sin\emptyset$$

- *Path of Approach depends on the dimensions of the driven wheel.*
- *Path of recess depends on the dimensions of the driving wheel (pinion)*





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Arc of Contact

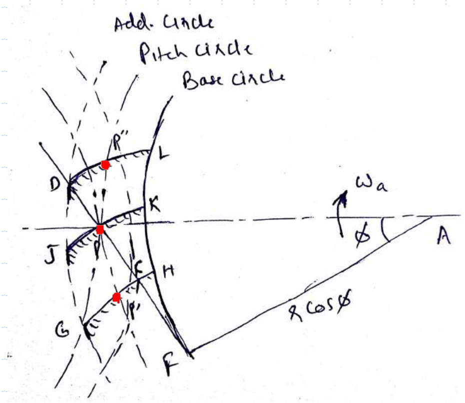
Arc of contact is the distance travelled by a point on either of the pitch circles of the two gears during the period of contact of a pair of teeth

GH: Driving involute at the beginning of contact

JK: Driving involute when the point of contact is at P.

DL: Driving involute at the end of engagement.

$P'P''$: Arc of contact = Arc of approach ($P'P$) + Arc of recess (PP'')



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Arc of Contact

t_a = time of approach

Arc of approach = $P'P$ = Tangential velocity of P' × Time of approach

$= \omega_a r \times t_a$

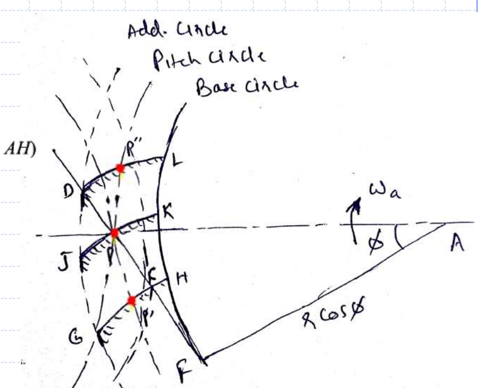
$= \omega_a (r\cos\varphi)\dfrac{1}{\cos\varphi} t_a$

$= (\text{Tang. vel. of } H) t_a \dfrac{1}{\cos\varphi}$ $\quad (AF = AH)$

$= \dfrac{\text{Arc } HK}{\cos\varphi}$

$= \dfrac{\text{Arc } FK - \text{Arc } FH}{\cos\varphi}$

$= \dfrac{FP - FC}{\cos\varphi} = \dfrac{CP}{\cos\varphi}$



Arc *FK* = Path *FP* as *P* is a point on generator *FP* that rolls on the base circle *FHK* to generate the involute *PK*.

Similarly, Arc *FH* = Path *FC*

Arc *of approach* $= \dfrac{\textit{Path of approach}}{\cos\phi}$

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Arc of Contact

t_r = time of recess

Arc of recess = PP'' = Tang. vel. of $P \times$ Time of recess

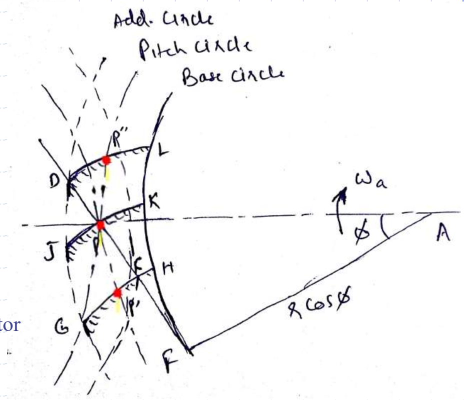
$= \omega_a r \times t_r$

$= \omega_a (r\cos\varphi)\dfrac{1}{\cos\varphi} t_r$

$= (\text{Tang. vel. of } K)\, t_r \dfrac{1}{\cos\varphi}$

$= \dfrac{\text{Arc } KL}{\cos\varphi} = \dfrac{\text{Arc } FL - \text{Arc } FK}{\cos\varphi}$

$PP'' = \dfrac{FD - FP}{\cos\varphi} = \dfrac{PD}{\cos\varphi}$



Arc FK = Path FP as P is a point on generator FP that rolls on the base circle FHK to generate the involute PK.
Similarly, Arc FL = Path FD

Arc of recess $= \dfrac{\text{Path of recess}}{\cos\emptyset}$

Arc of contact $= \dfrac{CP}{\cos\varphi} + \dfrac{PD}{\cos\varphi} = \dfrac{CP + PD}{\cos\varphi} = \dfrac{CD}{\cos\varphi}$

Arc of contact $= \dfrac{\text{Path of contact}}{\cos\varphi}$

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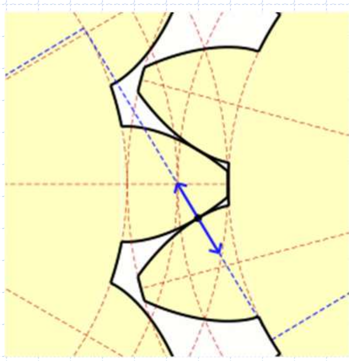
Contact Ratio

- The contact ratio of the gear set is defined as the ratio of length of arc of contact to circular pitch.
- This gives the average number of pairs of teeth in contact at a time.
- If contact ratio is more, the strength of gear set will be more.
- Contact ratio should always be greater than one.

Contact Ratio, $n = \dfrac{\text{Length of arc of contact}}{P_c}$



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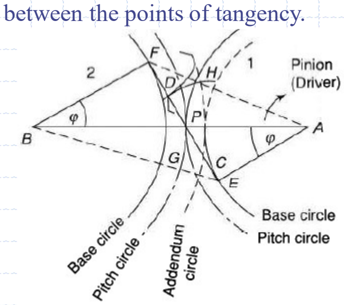
Interference

- The phenomenon when the tip of tooth undercuts the root of its mating gear is known as interference.
- The points E and F are called interference points.
- Interference may be avoided if the path of contact does not extend beyond interference points.
- The limiting value of the radius of the addendum circle of the pinion is *AF* and of the wheel is *BE*.
- Interference may only be avoided if the point of contact between the two teeth is always on the involute profile of the teeth. In other words, interference may only be prevented, if the addendum circle of the two mating gears cut the common tangent to the base circles between the points of tangency.



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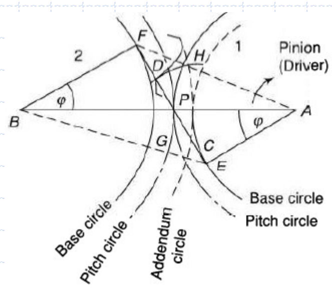
Interference

- When interference is just avoided,
  Maximum length of path of contact = EF
  = EP + PF
  Maximum length of path of approach, EP = $r \sin \emptyset$
  Maximum length of path of recess, PF = $R \sin \emptyset$
  $\therefore EF = (r + R) \sin \emptyset$
  And Maximum length of arc of contact = $\frac{(r+R)\sin \emptyset}{\cos \emptyset} = (r + R) \tan \emptyset$



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Minimum no. of teeth to avoid Interference

Maximum value of addendum radius of the wheel to avoid interference = *BE*

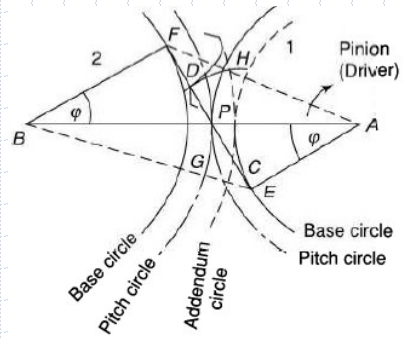
$$(BE)^2 = (BF)^2 + (FE)^2$$
$$= (BF)^2 + (FP + PE)^2$$
$$= (R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2$$
$$= R^2\left[1 + \frac{1}{R^2}(r^2 + 2rR)\sin^2\varphi\right]$$
$$BE = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi}$$

Maximum value of addendum of the wheel = (*BE* – Pitch circle radius)

$$a_{w\max} = R\left[\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi} - 1\right]$$

Now, $R = \frac{mT}{2}, r = \frac{mt}{2}$ and $G = \frac{T}{t}$ = Gear ratio

$$a_{w\max} = \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\varphi} - 1\right]$$
$$= \frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1\right]$$



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Minimum no. of teeth to avoid Interference

Let Addendum = $a_w\, m$

This value of addendum must be less than the maximum value to avoid interference.

$$\frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1\right] \geq a_w m$$

$$T \geq \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1}$$

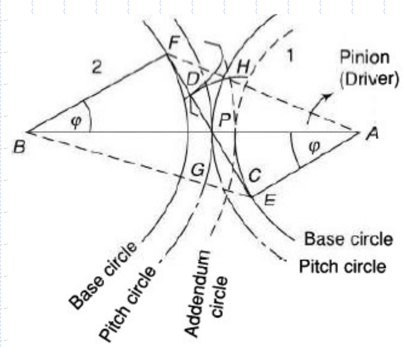
In the limit

$$T = \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1}$$

The minimum number of teeth on pinion is given by

$$t = \frac{T}{G}$$

For $a_w = 1$, $T \geq \frac{2}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1}$



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Minimum no. of teeth to avoid Interference

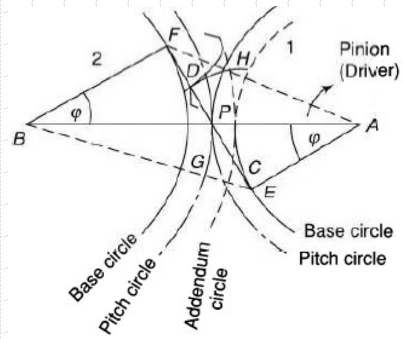
For $G = 1$,

$$T_{min} = \frac{2}{\sqrt{1+3\sin^2\varphi} - 1}$$

For $\emptyset = 20^{\circ}$,

$$T_{min} = \frac{2}{\sqrt{1+3\sin^2 20^{\circ}} - 1} = 12.31 \text{ or } 13$$

Thus for $G = 1$, $\emptyset = 20^{\circ}$, $a_w = 1$, the minimum number of teeth on each wheel must be 13 to avoid interference.



Maximum value of addendum radius of the wheel to avoid interference = AF

$$(AF)^2 = (r\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2$$

Maximum value of addendum of the pinion =

$$a_{p\max} = r\sqrt{1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\varphi} - r = \frac{mt}{2}\left[\sqrt{1+G(G+2)\sin^2\varphi} - 1\right]$$

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Interference between rack and pinion

Maximum value of addendum of the rack to avoid interference = GE

Let Addendum of rack = $a_r\ m$

Where a_r is the addendum coefficient

This value of addendum must be less than the maximum value to avoid interference.

$$GE = PE\sin\phi = (r\sin\phi)\sin\phi = r\sin^2\phi$$

$$= \frac{mt}{2}\sin^2\varphi$$

To avoid interference,

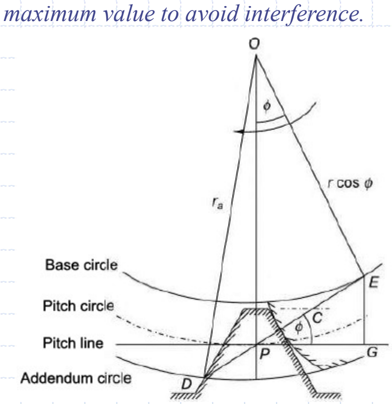
$$GE \geq a_r m \text{ or } \frac{mt}{2}\sin^2\varphi \geq a_r m \text{ or } t \geq \frac{2a_r}{\sin^2\varphi}$$

When a_r=1, i.e., for standard addendum,

$$t_{min} = \frac{2}{sin^2\emptyset}$$

For $\emptyset = 20^{\circ}$,

$$t_{min} = 17.1 \text{ or } 18$$



$$\text{Path of contact} = CP + DP = \frac{\text{Add. of rack}}{\cos\varphi} + \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$$

$$\text{Maximum path of contact to avoid interference} = DE = \sqrt{r_a^2 - (r\cos\varphi)^2}$$

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MEC2120
Kinematics of Machines

Gear Trains

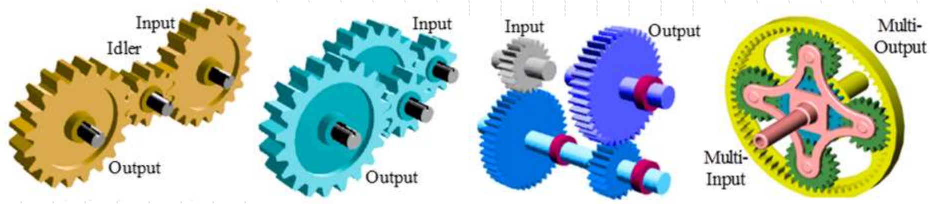
Gear Trains

When two or more gears are made to mesh with each other to transmit power from one shaft to another, the combination is called gear trains.

The nature of the train depends on velocity ratio and the relative position of the axes of shafts.

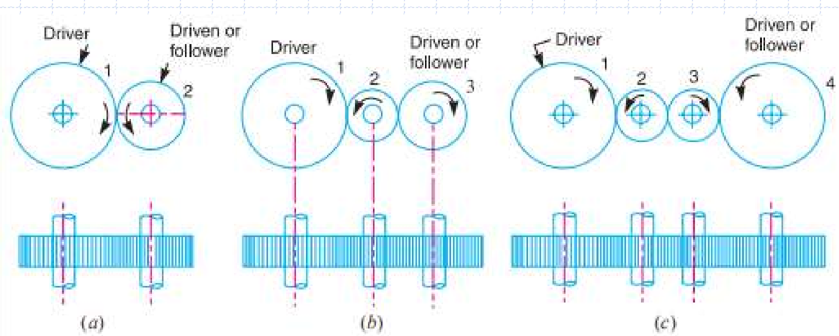
The gear train is classified into following types,

- Simple gear train.
- Compound gear train.
- Riveted gear train.
- Epicyclic gear train.

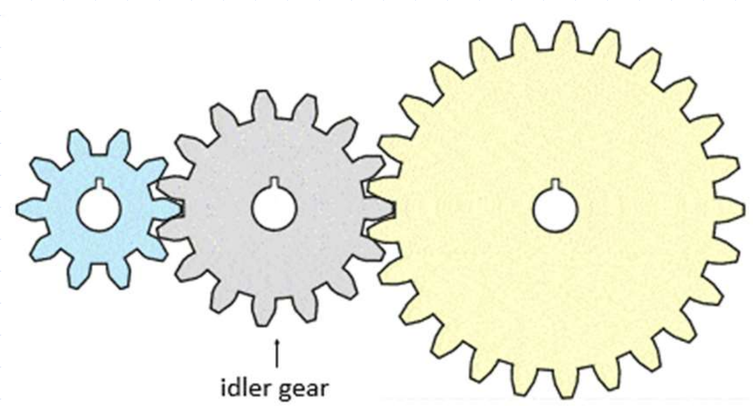


Simple Gear Train

When there is only one gear on each shaft, it is known as simple gear train. The gears are represented by their pitch circles.



Simple Gear Train



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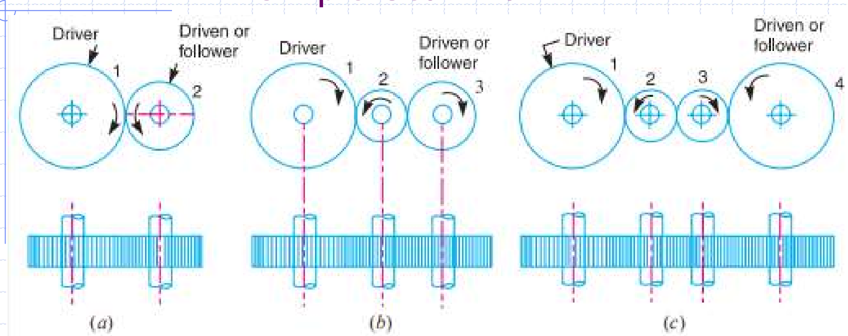
Simple Gear Train



In a Simple Gear Train

- Two external gears of a pair always move in opposite direction.
- All odd numbered gears move in one direction and all even numbered gears move in the opposite direction.
- When the number of intermediate gears are odd, the motion of both the gears (i.e. driver and driven or follower) is like.
- If the number of intermediate gears are even, the motion of the driven or follower will be in the opposite direction of the driver.

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Simple Gear Train

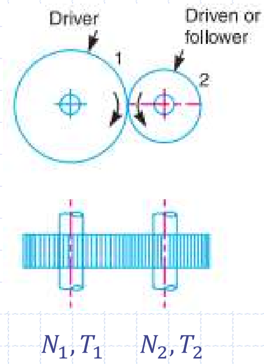
Speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth.

$$Speed\ Ratio = \frac{N_1}{N_2} = \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

Train Value of a Gear Train is the ratio of the speed of the driven or follower to the speed of the driver.

$$Train\ Value = \frac{N_2}{N_1} = \frac{\omega_2}{\omega_1} = \frac{T_1}{T_2}$$

$$Train\ Value = \frac{1}{Speed\ Ratio}$$



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Simple Gear Train

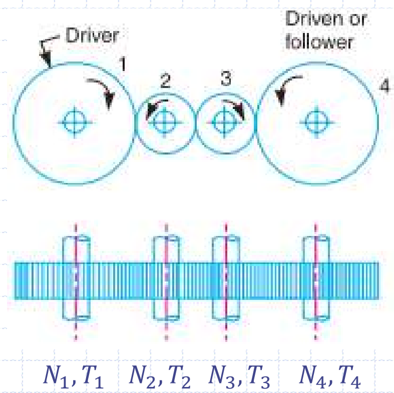
$$Speed\ Ratio = \frac{N_1}{N_4}$$

$$= \frac{N_1}{N_2} \times \frac{N_2}{N_3} \times \frac{N_3}{N_4} = \frac{N_1}{N_4}$$

$$= \frac{T_2}{T_1} \times \frac{T_3}{T_2} \times \frac{T_4}{T_3} = \frac{T_4}{T_1}$$

$$Speed\ Ratio = \frac{N_1}{N_4} = \frac{T_4}{T_1}$$

$$Train\ Value = \frac{N_4}{N_1} = \frac{\omega_4}{\omega_1} = \frac{T_1}{T_4}$$



- In a **Simple Gear Train**, the **Speed Ratio and Train value** are independent of the size and number of intermediate gears.
- These intermediate gears are called idle gears, as they do not effect the speed ratio or train value of the system.

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Compound Gear Train

When there are more than one gear on a shaft, it is called a Compound Gear train.

- For Gear Pair (Gear 1 and 2)

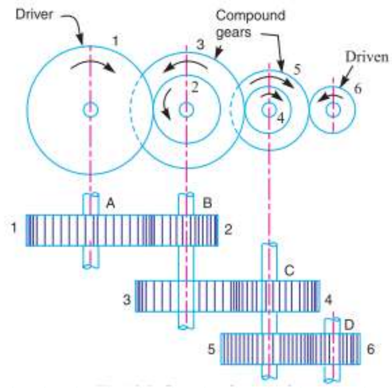
$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \text{- - - -(i)}$$

- For Gear Pair (Gear 3 and 4)

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \qquad \text{- - - -(ii)}$$

- For Gear Pair (Gear 5 and 6)

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \qquad \text{- - - -(iii)}$$



The speed ratio of compound gear train

$$\frac{N_1}{N_6} = \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} \qquad \left\{ \because N_2 = N_3, \; N_4 = N_5 \right\}$$

$$= \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$Gear\ 1\!: N_1, T_1$ $\quad Gear\ 5\!: N_5, T_5$

$Gear\ 2\!: N_2, T_2$ $\quad Gear\ 6\!: N_6, T_6$

$Gear\ 3\!: N_3, T_3$

$Gear\ 4\!: N_4, T_4$

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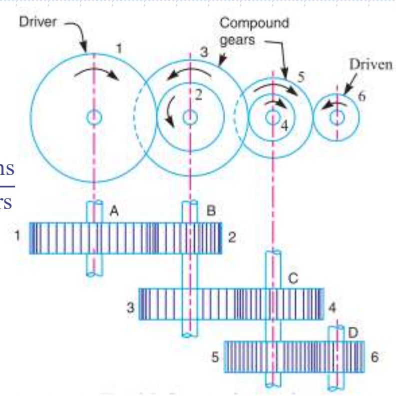
Compound Gear Train

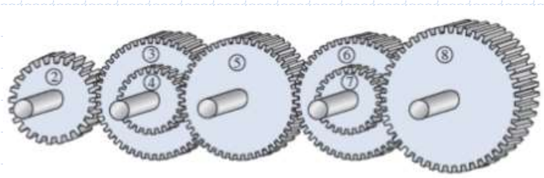
In a Compound Gear Train

$$Speed\ Ratio = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

- The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.





$Gear\ 1\!: N_1, T_1$ $\quad Gear\ 5\!: N_5, T_5$

$Gear\ 2\!: N_2, T_2$ $\quad Gear\ 6\!: N_6, T_6$

$Gear\ 3\!: N_3, T_3$

$Gear\ 4\!: N_4, T_4$

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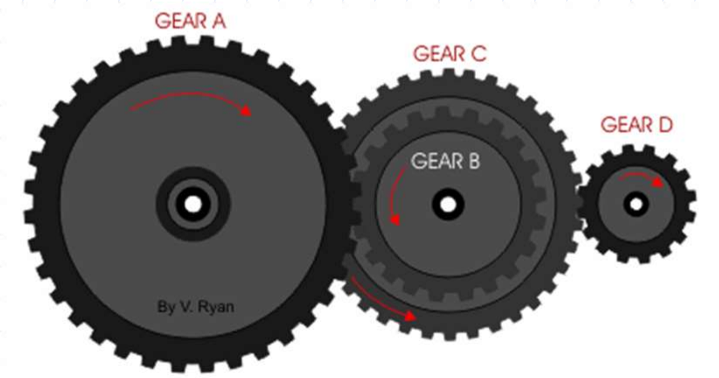


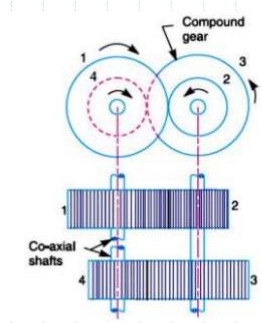
Reverted Gear Train

- When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train

$$C = r_1 + r_2 = r_3 + r_4$$

$$\frac{N_4}{N_1} = \frac{\text{Product of number of teeth on the drivers}}{\text{Product of number of teeth on the drivens}}$$

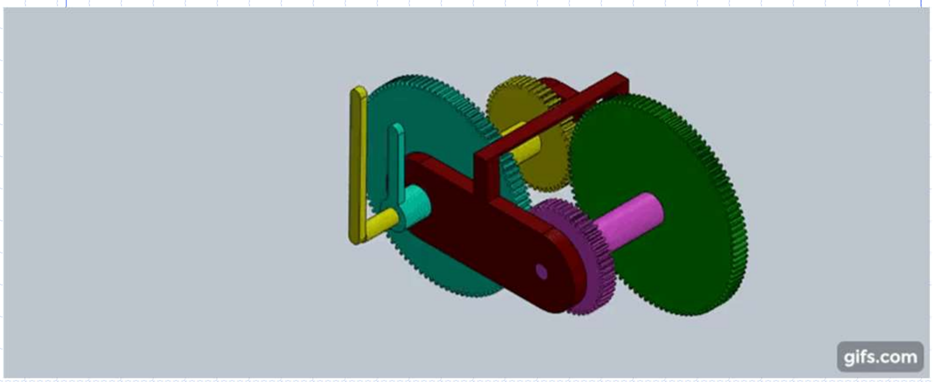
$$\frac{N_4}{N_1} = \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$



- For the same circular pitch or module of all the gears

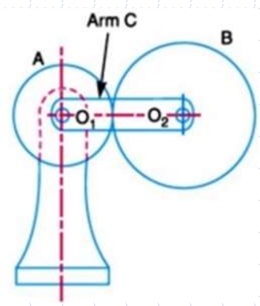
$$T_1 + T_2 = T_3 + T_4$$

- The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and clocks.



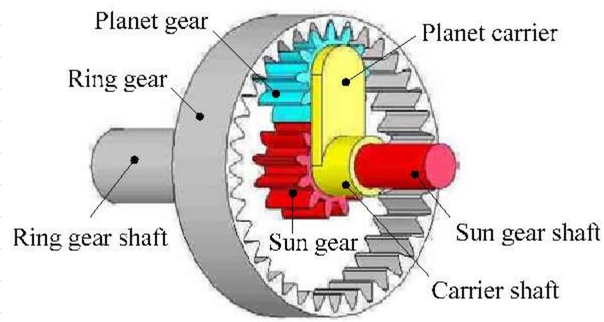
Epicyclic Gear Train

- In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis.
- The ***upon*** and ***around*** motion is called epicyclic motion (***epi*** - means upon and ***cyclic*** means around).
- The gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains*** .
- The epicyclic gear trains may be ***simple or compound.***



- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space.
- The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

Epicyclic Gear Train



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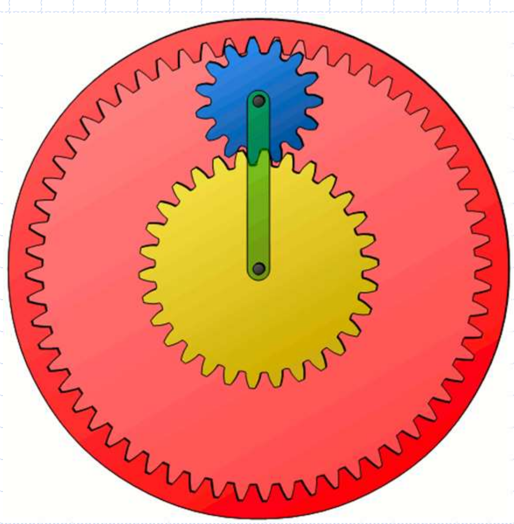
Epicyclic Gear Train



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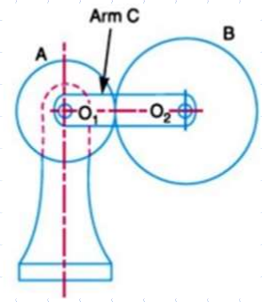
Analysis of Epicyclic Gear Train

Tabulation Method

CONDITION OF MOTION	ARM C	GEAR A	GEAR B
The arm is fixed, Gear A rotates through +1 revolution	0	+1	-*Ta/Tb*
Arm fixed Gear A rotates through +*x* revolution	0	+*x*	-*x* × *Ta/Tb*
Add +*y* to all elements	+*y*	+*y*	+*y*
Total Motion	*y*	*x* + *y*	*y* - *x* × *Ta/Tb*



- Clockwise rotation is assumed as positive and Anticlockwise as negative.

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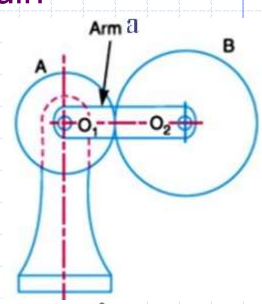
Analysis of Epicyclic Gear Train

Problem: In an epicyclic gear train, an arm carries two gears A and B having 30 and 40 teeth respectively. If the arm rotates at 80 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 240 r.p.m. in the clockwise direction, what will be the speed of gear B ?



Solution: Considering CCW rev. as + ve.

CONDITION OF MOTION	ARM a	GEAR A	GEAR B
The arm is fixed, Gear A rotates through +1 revolution	0	+1	-*Ta/Tb*
Arm fixed Gear A rotates through +*x* revolution	0	+*x*	-*x* × *Ta/Tb*
Add +*y* to all elements	+*y*	+*y*	+*y*
Total Motion	*y*	*x* + *y*	*y* - *x* × *Ta/Tb*

(i) Gear *A* is fixed, thus $y + x = 0$

Arm *a* rotates at 80 rpm, $y = 80$

$\therefore$ $x = -80$

Speed of the gear *B*, $y - \frac{3}{4}x$

$80 - \frac{3}{4} \times (-80) = 140 =$ rpm (counter-clockwise)

(i) Gear *A* revolves at 240 rpm clockwise,

$y + x = -240$

$\therefore \quad x = -80 - 240 = -320$

Speed of the gear *B*, $y - \frac{3}{4}x$

$= 80 - \frac{3}{4} \times (-320)$

$= 320$ rpm (counter-clockwise)

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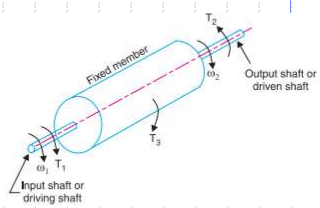
Torques in Epicyclic Gear Train

When the rotating parts of an epicyclic gear train have no angular acceleration, the gear train is kept in equilibrium by the three externally applied torques, viz.

1. Input torque on the driving member (T1),
2. 2. Output torque or resisting or load torque on the driven member (T2),
3. 3. Holding or braking or fixing torque on the fixed member (T3)



The net torque applied to the gear train must be zero.
In other words, T1 + T2 + T3 = 0

Let $\omega_1, \omega_2, \omega_3$ be the angular speeds of the driving, driven and fixed members respectively, and the friction be neglected, then the net kinetic energy dissipated by the gear train must be zero,
i.e.
$T_1\omega_1 + T_2\omega_2 + T_3\omega_3 = 0$...(iii)
But, for a fixed member, $\omega_3 = 0$
$\therefore T_1\omega_1 + T_2\omega_2 = 0$...(iv)
$\Rightarrow T_2 = -T_1 \times \frac{\omega_1}{\omega_2}$

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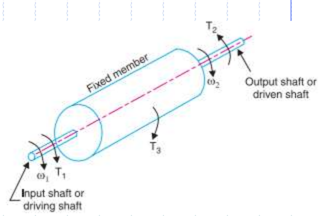
Torques in Epicyclic Gear Train

$$T_2 = -T_1 \times \frac{\omega_1}{\omega_2}$$

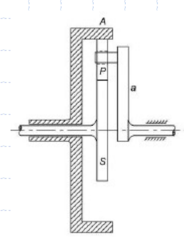
The above Equation shows that when input shaft (or driving shaft) and output shaft (or driven shaft) rotate in the same direction, then the input and output torques will be in opposite directions. Similarly, when the input and output shafts rotate in opposite directions, then the input and output torques will be in the same direction.



$$T_3 = -(T_1 + T_2)$$

$$T_3 = T_1\left(\frac{\omega_1}{\omega_2} - 1\right)$$

$$T_3 = T_1\left(\frac{N_1}{N_2} - 1\right)$$



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