MEC2120
Kinematics of Machines

Unit 2

Kinematic Analysis

- Kinematics is the study of displacement, rotation, speed, velocity and acceleration of each link at various positions during the operating cycle.
- Using these information, a designer can compute forces and thereby dimensions of all the links.
- In kinematic analysis, the motion of a point on the link relative to the fixed frame of reference is called absolute motion. However, the motion of a point relative to some other moving link or moving frame of reference is said to be relative motion.

Velocity Analysis

- The change of position of a link with reference to some fixed frame of coordinates is called displacement.
- The rate of change of displacement of a link with reference to time, i.e. the time derivative of displacement, is commonly referred as velocity of link.
- Depending upon the type of motion, the velocity is classified into two types, namely linear velocity and angular velocity.

Linear velocity: $v = \frac{ds}{dt}$

Angular velocity: $\omega = \frac{d\theta}{dt}$

In kinematic analysis, the velocities of various links can be determined by the following methods:

(i) Relative velocity method or velocity polygon method
(ii) Instantaneous center method
(iii) Analytical method

Relative velocity method

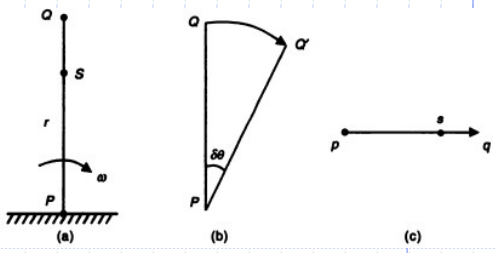
$$v_{qp} = \frac{\delta\theta}{\delta t} \times r$$

$$v_{qp} = \frac{d\theta}{dt} \times r \quad \text{when } \delta t \to 0$$

$$v_{qp} = \omega r$$

$$v_{sp} = \omega \cdot SP$$

$$\frac{v_{sp}}{v_{qp}} = \frac{\overline{ps}}{\overline{pq}} = \frac{\omega \times SP}{\omega \times QP} = \frac{SP}{QP}$$



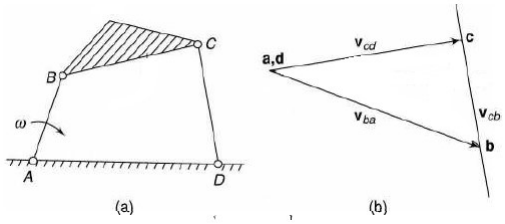
- Hence the point s divides the vector pq in the same ratio as the point S divides the link PQ.
- Thus, this law of proportionality is useful in drawing the velocity polygon and finding the relative velocities of points on the link.

Four Link Mechanism

Four Bar Mechanism

AD: Fixed Link, AB: Crank, CD: Rocker

ω: Angular Speed of AB (Clockwise)



$\vec{V}_{ca}=\vec{V}_{ba}+\vec{V}_{cb}$ or $\vec{V}_{cd}=\vec{V}_{ba}+\vec{V}_{cb}$ or $\overrightarrow{dc}=\overrightarrow{ab}+\overrightarrow{bc}$

Where, $\vec{V}_{ba}$ or $\overrightarrow{ab} = \omega AB$; $\perp$ to AB

$\vec{V}_{cb}$ or $\overrightarrow{bc}$ is unknown in magnitude; $\perp$ to BC

$\vec{V}_{cd}$ or $\overrightarrow{dc}$ is unknown in magnitude; $\perp$ to DC

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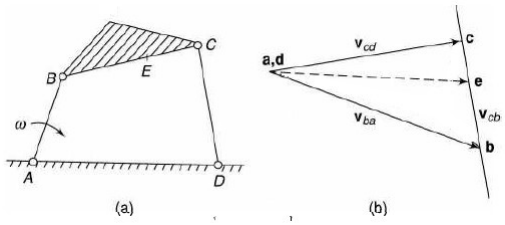
Four Link Mechanism

- **Four Bar Mechanism**

Velocity of Intermediate Point (Point E on Link BC)



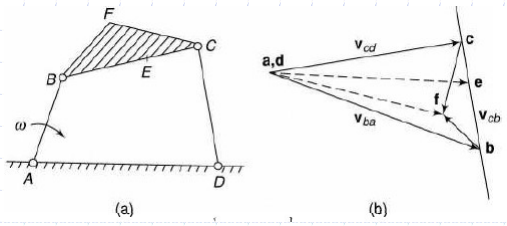
$$\frac{be}{bc} = \frac{BE}{BC}$$

Here, $\overrightarrow{ae}$: *Absolute Velocity of Point E on Link BC*

Four Link Mechanism

- **Four Bar Mechanism**

Velocity of Offset Point (Point F on Link BC)



$$\vec{V}_{ba} + \vec{V}_{fb} = \vec{V}_{cd} + \vec{V}_{fc}$$

$$\overrightarrow{ab} + \overrightarrow{bf} = \overrightarrow{dc} + \overrightarrow{cf}$$

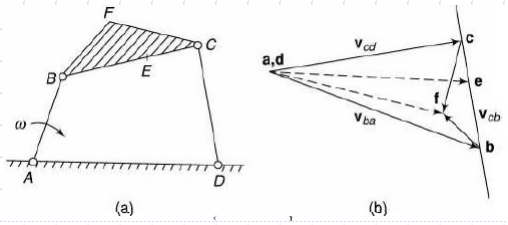
$\vec{V}_{fb} \perp$ **to** BF $\qquad$ $\vec{V}_{fc} \perp$ **to** CF

Here, $\overrightarrow{af}$: *Absolute Velocity of Point F on Link BC*

Four Link Mechanism

- **Four Bar Mechanism**

Velocity of Offset Point (Point F on Link BC)



Velocity Image:

$\triangle bfc \sim \triangle BFC$

Where,

$bf \perp$ to BF $\quad fc \perp$ to FC and $cb \perp$ to CB

Here, $\triangle bfc$ is called Velocity Image

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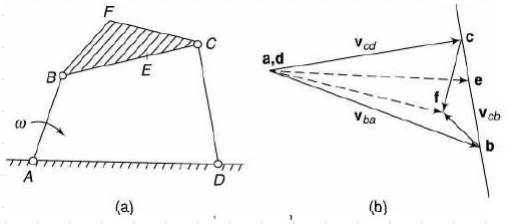
Four Link Mechanism

- **Four Bar Mechanism**

Angular Velocity of Links



- **Link BC**

$\omega_{CB} = \frac{v_{cb}}{CB}$ **(C.C.W)**

- **Link CD**

$\omega_{CD} = \frac{v_{cd}}{CD}$ **(C.W)**

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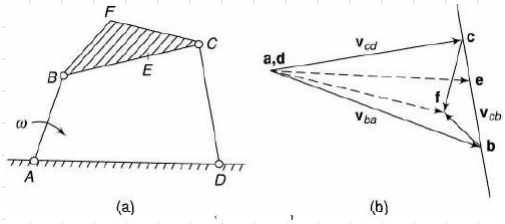
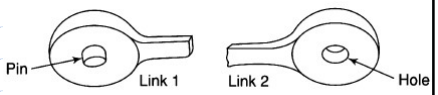
Four Link Mechanism

- **Four Bar Mechanism**

Velocity of Rubbing



- **Pin at A**
  Velocity of Rubbing $= r_A\omega_{AB} = r_A\omega$
- **Pin at D**
  Velocity of Rubbing $= r_D\omega_{CD}$
- **Pin at B**
  Velocity of Rubbing $= r_B(\omega_{AB} + \omega_{BC})$
- **Pin at C**
  Velocity of Rubbing $= r_C(\omega_{BC} + \omega_{CD})$



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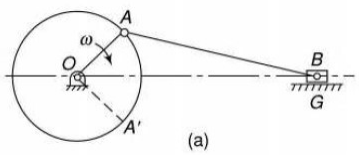
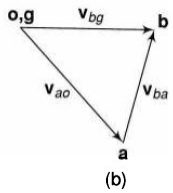
Four Link Mechanism

- **Slider Crank Mechanism**

OG: Fixed Link, OA: Crank, AB: Coupler, B: Slider
ω: Angular Speed of OA (Clockwise)





$\vec{V}_{bo}=\vec{V}_{ao}+\vec{V}_{ba}$ $\qquad$ $\overrightarrow{ob}=\overrightarrow{oa}+\overrightarrow{ab}$

Where, $\vec{V}_{ao}$ or $\overrightarrow{oa} = \omega OA$; ⊥ to OA

$\vec{V}_{ba}$ or $\overrightarrow{ab}$ is unknown in magnitude; ⊥ to AB

$\vec{V}_{bo}$ or $\overrightarrow{ob}$ is unknown in magnitude; ∥ to the motion of B

Angular Velocity of Link AB: $\omega_{AB} = \frac{v_{ba}}{AB}$ (C.C.W)

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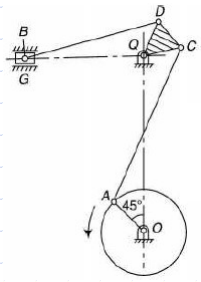
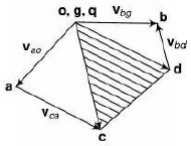
Problem 1.





OA = 150 mm
AC = 600 mm
CQ = QD = 145 mm

CD = 125 mm
BD = 500 mm
OQ = 625 mm

$N_{OA} = 60rpm$ (c.c.w)

$\vec{V}_{bo} = ?$, $\omega_{BD} = ?$

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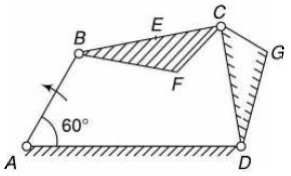
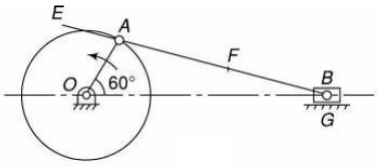
Problem 2.



Problem 3.



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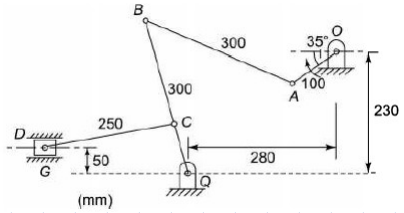
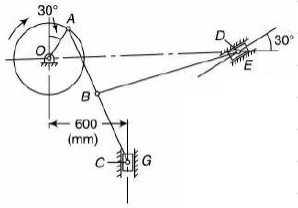
Problem 4.



Problem 5.



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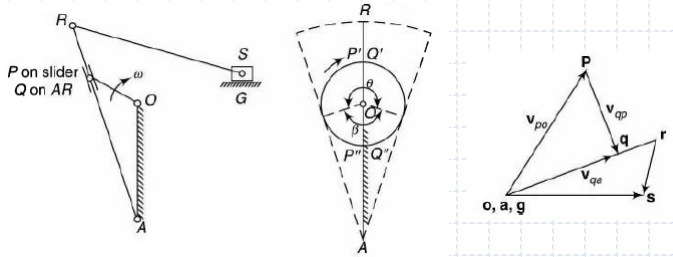
Crank and Slotted Lever Mechanism

OA: Fixed Link, OP: Crank, P, S: Slider, AR: Oscillating Link

***ω*: *Angular Speed of OP* (*Clockwise*)**



$\vec{V}_{qo}=\vec{V}_{po}+\vec{V}_{qp}$ or $\vec{V}_{qa}=\vec{V}_{po}+\vec{V}_{qp}$ or $\overrightarrow{aq}=\overrightarrow{op}+\overrightarrow{pq}$

Where, $\vec{V}_{po}$ or $\overrightarrow{op} = \omega OP$; ⊥ to *OP*

$\vec{V}_{qp}$ or $\overrightarrow{pq}$ is unknown in magnitude; ∥ to *AR*

$\vec{V}_{qa}$ or $\overrightarrow{aq}$ is unknown in magnitude; ⊥ to *AR*

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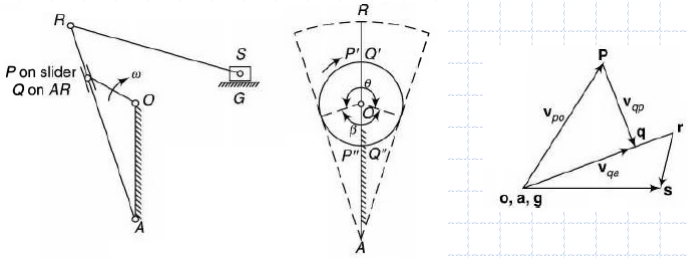
Crank and Slotted Lever Mechanism



$\vec{V}_{ra} \perp$ to AR

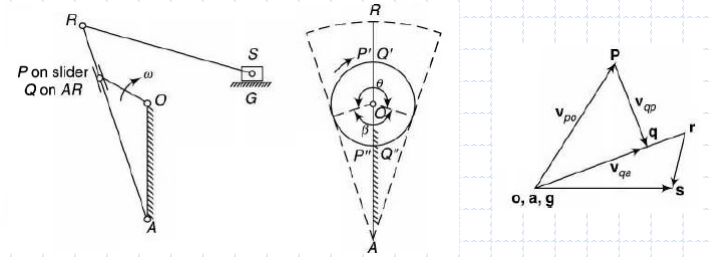
r will lie on $\overrightarrow{aq}$ Produced such that $ar/aq = AR/AQ$

$\vec{V}_{sa}=\vec{V}_{ra}+\vec{V}_{sr}$ $\quad \overrightarrow{as}=\overrightarrow{ar}+\overrightarrow{rs}$

$\vec{V}_{sr}$ or $\overrightarrow{rs}$ is unknown in magnitude; $\perp$ to RS

$\vec{V}_{sa}$ or $\overrightarrow{as}$ is unknown in magnitude; $\parallel$ to the motion of S

Crank and Slotted Lever Mechanism



$$\frac{\text{Time of cutting}}{\text{Time of Return}} = \frac{\theta}{\beta}$$

At P', during cutting stroke, $pq=0$ and oq is maximum$= op$

Let

r = length of crank ($= OP$)

l = length of slotted lever ($= AR$)

c = distance between fixed centres ($= AO$)

ω = angular velocity of the crank

during the cutting stroke,

$$v_{s\,\max} = \omega \times OP' \times \frac{AR}{AQ} = \omega r \times \frac{l}{c+r}$$

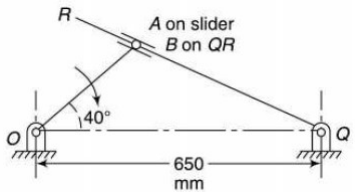
during the return stroke,

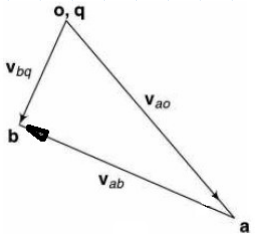
$$v_{s\,\max} = \omega \times OP'' \times \frac{AR}{AQ''} = \omega r \times \frac{l}{c-r}$$

$$\frac{v_{s\,\max}\,(\text{cutting})}{v_{s\,\max}\,(\text{return})} = \frac{\omega r \dfrac{1}{c+r}}{\omega r \dfrac{1}{c-r}} = \frac{c-r}{c+r}$$

Crank and Slotted Lever Mechanism

Problem 1.





$\vec{V}_{bq}=\vec{V}_{ao}+\vec{V}_{ba}$ $\quad \overrightarrow{qb}=\overrightarrow{oa}+\overrightarrow{ab}$

Where, $\vec{V}_{ao}$ **or** $\overrightarrow{oa} = \omega OA$; $\perp$ **to** OA

$\vec{V}_{ba}$ **or** $\overrightarrow{ab}$ **is unknown in magnitude;** $\parallel$ **to** QR

$\vec{V}_{bq}$ **or** $\overrightarrow{qb}$ **is unknown in magnitude;** $\perp$ **to** QR

$$\omega_{qr} = \omega_{qb} = \frac{v_{qb} \text{ or } v_{bq}}{BQ}$$

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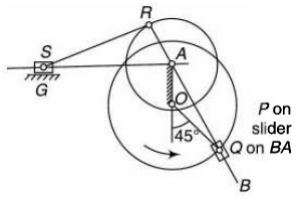
Crank and Slotted Lever Mechanism

Problem 2.



$OP = 240\ mm, OA = 150\ mm, AR = 165\ mm$

$RS = 430\ mm, \omega_{op} = 2.5\ rad/s$

(1) velocity of the ram S**.**

(2) velocity of the slider P **on the slotted lever.**

(3) angular velocity of the link RS

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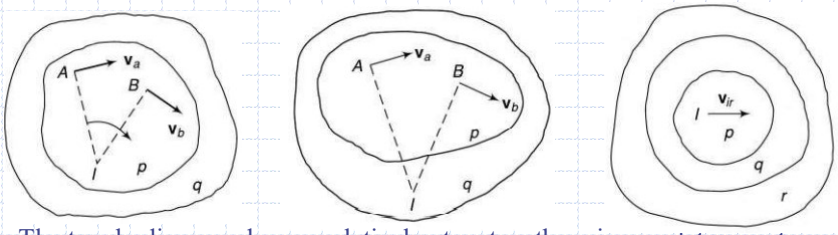
Instantaneous Centre

- If the displacement of a rigid body having plane motion is considered as equivalent to a pure rotation of the body as a whole about some centre, such centre is called instantaneous centre.
- Any point on a rigid body or on its extension that has zero velocity is called the Instantaneous Center of Velocity of the body.



- The two bodies p and q are relatively at rest or there is no relative motion between the two at the I-centre.
- If body q is also in motion relative to a third body r, then the motion of the point I relative to the third body would be the same whether this point is considered on the body p or q.

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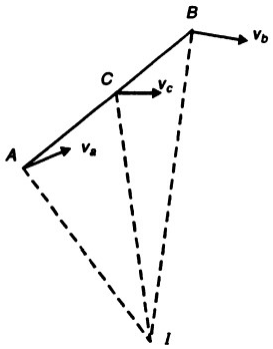
Instantaneous Centre

- Velocity of any point in the body is proportional to its distance from the Instantaneous centre
- The direction of velocity is in a line perpendicular to the line joining that point to the Instantaneous centre.



$$\frac{v_c}{IC} = \frac{v_a}{IA} = \frac{v_b}{IB} = \text{angular velocity of link } AB$$

- Thus an Instantaneous centre makes it possible to determine the velocity of any point on a link if velocities of at least two points on the link or location of its instantaneous centre are known.

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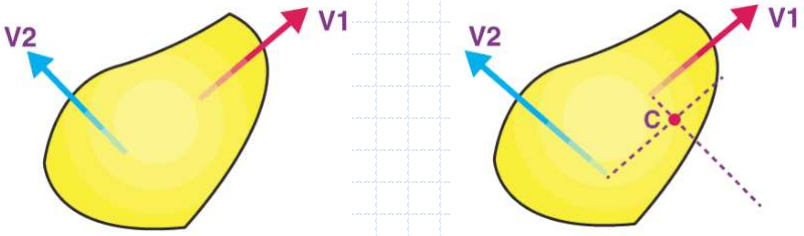
Instantaneous Centre

How can we locate the instantaneous centre of rotation?

Case I : When two arbitrary (Non Parallel) velocities are given:



- Draw a perpendicular from the tails of two velocity vectors, the point where they intersect is the ICOR.

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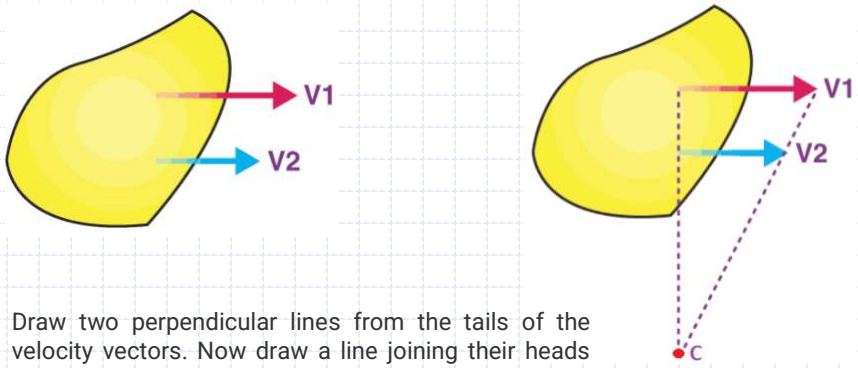
Instantaneous Centre

How can we locate the instantaneous centre of rotation?

Case II: When two Parallel velocities are given in the same direction as shown:



- Draw two perpendicular lines from the tails of the velocity vectors. Now draw a line joining their heads which intersects the perpendicular line. That point of intersection is the ICOR.

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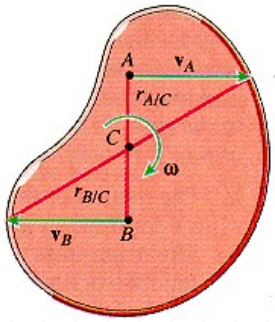
Instantaneous Centre

How can we locate the instantaneous centre of rotation?

Case III: When two Parallel velocities are given in the opposite direction as shown:



- Draw two perpendicular lines from the tails of the velocity vectors. Now draw a line joining their heads which intersects the perpendicular line. That point of intersection is the ICOR.

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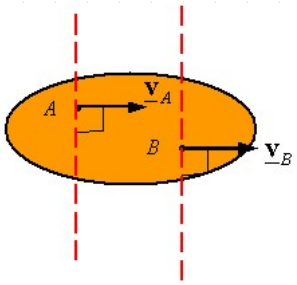
Instantaneous Centre

How can we locate the instantaneous centre of rotation?

Case IV: When two Parallel and equal velocities are given in same direction as shown:



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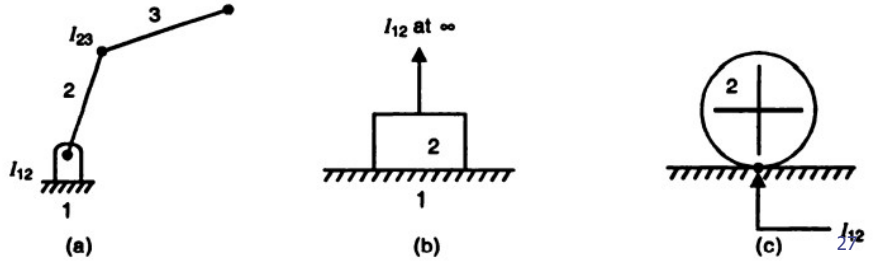
Instantaneous Centre

- Instant center of velocities is a simple graphical method for performing velocity analysis on mechanisms.
- The method provides visual understanding on how velocity vectors are related.
- The location of instantaneous centres for various types of motion in a mechanism can be decided on the basis of the following rules:
  1. When two links form a turning pair, the instantaneous centre is assumed to be located at the centre of the pair [Figure (a)].
  2. In case of sliding pair, the instantaneous centre lies at infinity in the direction perpendicular to the path of motion of the slider [Figure (b)].
  3. When two links make a pure rolling contact, the point of contact at a given instant is taken as instantaneous centre [Figure (c)].



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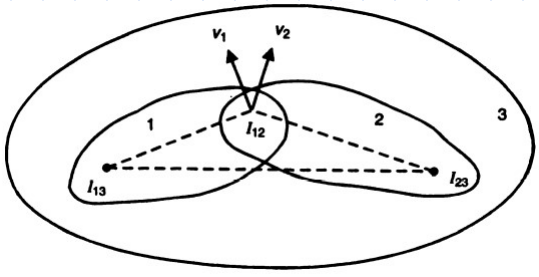
Arnold Kennedy Theorem

- If three links have relative motion with respect to each other, their relative instantaneous centre lies on a straight line.

or

- The three instant centers between three planar links must lie on a straight line.



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Locating Instantaneous Centres

Determine the Number of Instantaneous Centres

In a mechanism with n links (count the ground as one of the links), the number of instant centers is determined as:

$$C = n(n-1)/2$$

e.g.: In a four-bar mechanism or a slider-crank, there are six IC's (n = 4).
For any six-bar mechanism, C = 15 .

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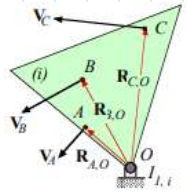
Locating Instantaneous Centres

Instant Center Between a Link and The Ground

a) link i that is pinned to the ground at O

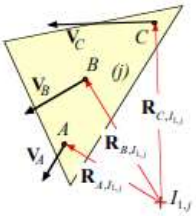
The velocity of any point on the link is determined as

$$\mathbf{V} = \boldsymbol{\omega}\mathbf{R}$$

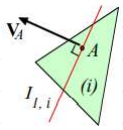


b) link j that is not connected to the ground directly

$$V_A / R_{A,I_{1,j}} = V_B / R_{B,I_{1,j}} = V_C / R_{C,I_{1,j}} = \omega$$



Note: If we know the velocity (absolute) of a point on a link, the instant center between that link and the ground must be located on an axis perpendicular to the velocity vector passing through the point.



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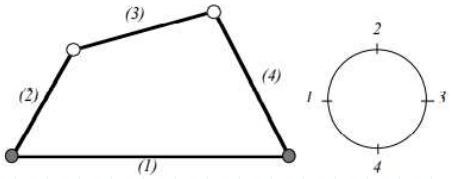
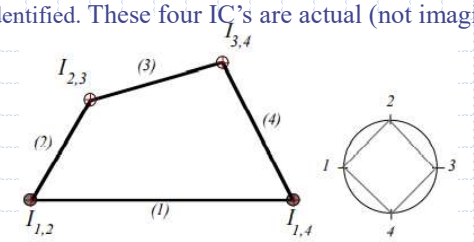
Locating Instantaneous Centres

Instant Centers of A Four-bar Mechanism

A four-bar mechanism has six instant centers regardless of the dimensions or orientation of the links.



Pin joints are instant centers, for a four bar with four pin joints, four IC's are immediately identified. These four IC's are actual (not imaginary) pin joints.



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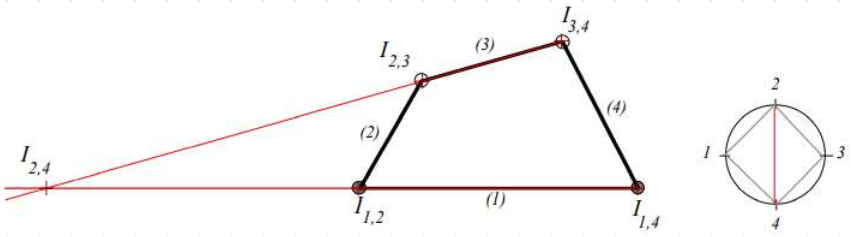
Locating Instantaneous Centres

Instant Centers of A Four-bar Mechanism

In order to find the other IC's, we apply Kennedy's rule.

The IC's between links 2, 3 and 4 must lie on a straight line. These are I_{23} , I_{34} , and I_{24} .

The IC's between links 1, 2 and 4 must lie on a straight line. These are I_{12} , I_{14} , and I_{24} .



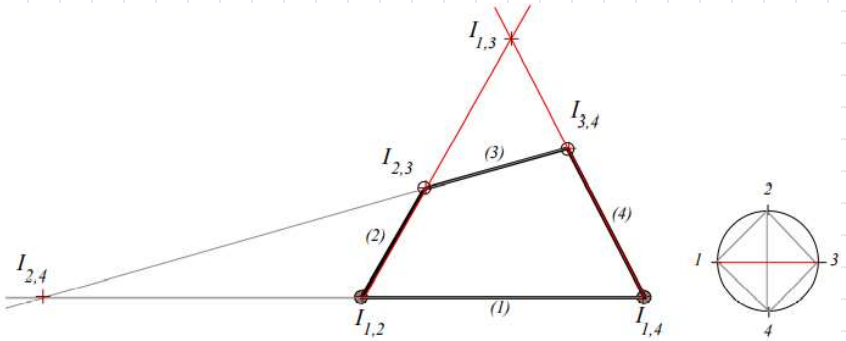
32

Locating Instantaneous Centres

Instant Centers of A Four-bar Mechanism

In order to find the other IC's, we apply Kennedy's rule.

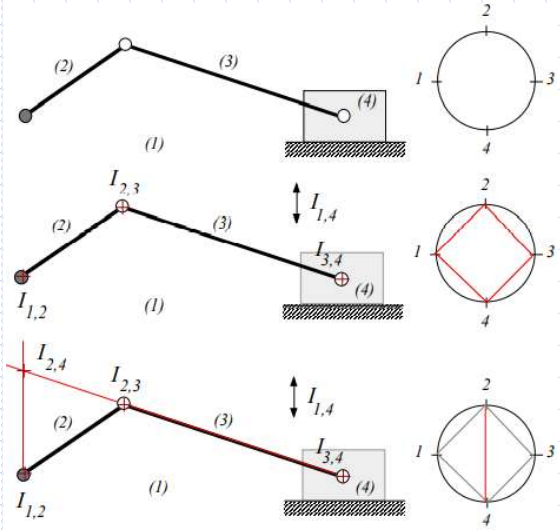
The last center to find is between links 1 and 3. The IC's between links 1, 2 and 3 must lie on a straight line. These are I_{12} , I_{23} , and I_{13} .
The IC's between links 1, 4 and 3 must lie on a straight line. These are I_{14} , I_{34} , and I_{13} .



33

Locating Instantaneous Centres

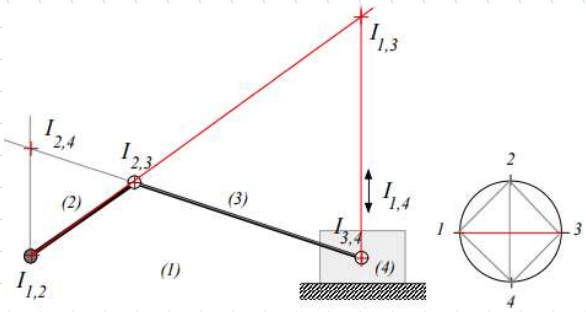
Instantaneous Centers of A Slider Crank Mechanism Mechanism



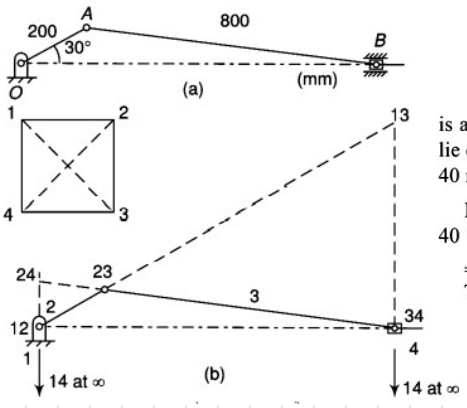
34

Locating Instantaneous Centres

Instantaneous Centers of A Slider Crank Mechanism Mechanism



35



The point 24 has the same velocity whether it is assumed to lie in link 2 or 4. First, assume 24 to lie on the link 2 which rotates at angular velocity of 40 rad/s.

Linear velocity of I-centre 24 = 40 × (12–24) = 40 × 0.123

= 4.92 m/s in the horizontal direction

Thus, linear velocity of the slider = 4.92 m/s

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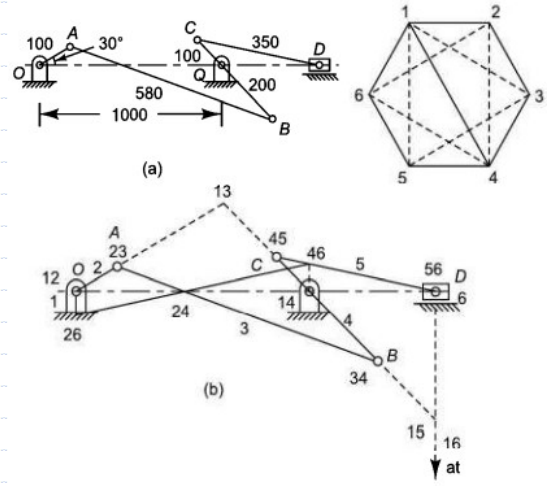


Now, at the instance, the link 3 rotates about the I-centre 13.

$$\frac{v_b}{v_a} = \frac{13-34}{13-23} \text{ or } v_b = \frac{453}{265} \times 1.57 = 2.66 \text{ m/s}$$

$$\frac{v_c}{v_b} = \frac{QC}{QB} \text{ or } v_c = \frac{100}{200} \times 2.66 = 1.33 \text{ m/s}$$

At the instance, the link 5 rotates about the I-centre 15.

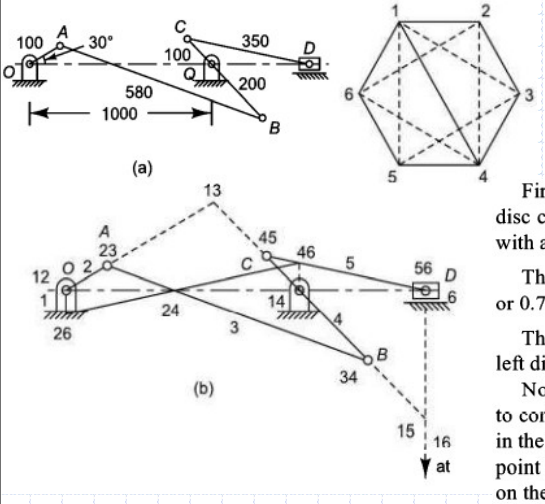
Thus, $\frac{v_d}{v_c} = \frac{15-56}{15-45}$ or

$$v_d = \frac{300}{506} \times 1.33 = 0.788 \text{ m/s}$$

$$\omega_{ab} = \frac{v_a}{13-A} = \frac{1.57}{0.267} = 5.88 \text{ rad/s}$$

$$\omega_{bc} = \frac{v_b}{BQ} = \frac{2.66}{0.2} = 13.3 \text{ rad/s}$$

$$\omega_{cd} = \frac{v_c}{15-c} = \frac{1.33}{0.499} = 2.66 \text{ rad/s}$$



First, imagine the link 2 to be in the form of a flat disc containing the point 26 and revolving about O with an angular velocity of 15.7 rad/s.

Then, $v_{26} = \omega_2 \times (12\text{–}26) = 15.7 \times 50 = 785$ mm/s or 0.785 m/s

The velocity of the point 26 is in the horizontally left direction if OA rotates clockwise.

Now, imagine the link 6 (slider) to be large enough to contain the point 26. The slider can have motion in the horizontal direction only and the velocity of a point 26 on it is known; it implies that all the points on the slider move with the same velocity.

Thus, velocity of the slider, $v_d = v_{26} = \underline{785 \text{ mm/s}}$

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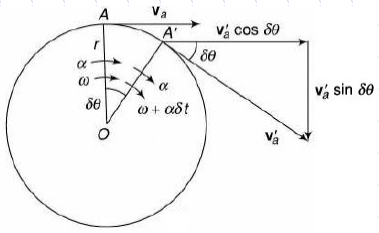
Acceleration analysis
(Graphical Method)

Acceleration



Angular velocity of $OA = \omega$

Tangential velocity of A, $v_a = \omega r$

Angular velocity of OA', $\omega'_a = \omega + \alpha \delta t$

Tangential velocity of A', $v'_a = (\omega + \alpha \delta t) r$

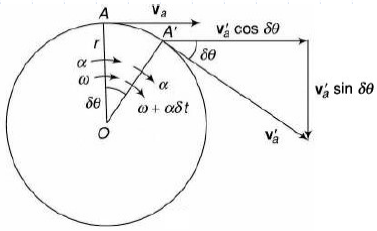
Change of Velocity Perpendicular to OA

$$\text{Acceleration of } A \perp \text{to } OA = \frac{v'_a \cos \delta\theta - v_a}{\delta t}$$

$$\text{Acceleration of } A \perp \text{to } OA = \frac{(\omega + \alpha . \delta t) r \cos \delta\theta - \omega r}{\delta t}$$

2

Acceleration



Acceleration of $A \perp$ to $OA = \alpha r = \left(\frac{d\omega}{dt}\right) r = \frac{dv}{dt}$ $\quad \ldots \left(\alpha = \frac{d\omega}{dt}\right)$ In the limit, as $\delta t \to 0$, $\sin \delta\theta \to \delta\theta$

This is known as Tangential acceleration of A relative to O.

$f^t_{ao} = \frac{dv}{dt}$

Change of Velocity Parallel to OA

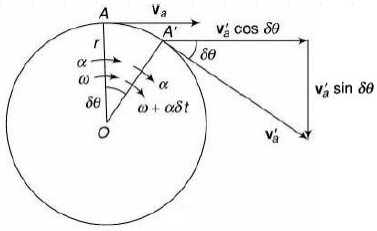
Acceleration of $A \parallel$ to $OA = \frac{v'_a \sin \delta\theta - 0}{\delta t}$

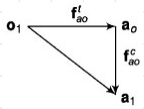
Acceleration of $A \parallel$ to $OA = \frac{(\omega + \alpha\delta t) r \sin \delta\theta}{\delta t}$

Acceleration of $A \parallel$ to $OA = \omega^2 r = \frac{v^2}{r}$ In the limit, as $\delta t \to 0$, $\sin \delta\theta \to \delta\theta$

This is known as Centripetal or Radial acceleration of A relative to O.

Acceleration



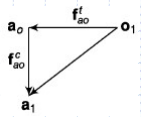


$f^c_{ao} = \omega^2 r = \frac{v^2}{r}$

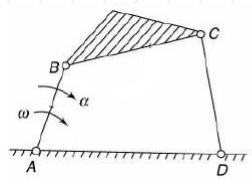
When $\alpha = 0$, f^t_{ao}=0 and f^c_{ao} represents the total acceleration.

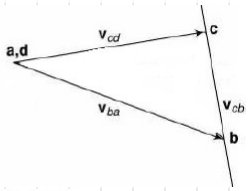
When $\omega = 0$, f^c_{ao}=0 and f^t_{ao} represents the total acceleration.

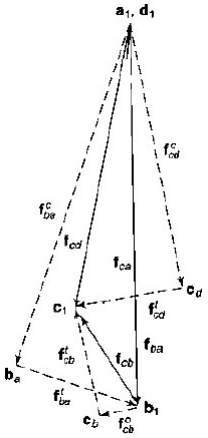
When α is negative, f^t_{ao} is negative.



Four Link Mechanism
4R Planar Mechanism







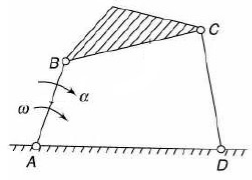
$\mathbf{f_{ca} = f_{cd} = f_{ba} + f_{cb}} \qquad \mathbf{d_1c_1 = a_1b_1 + b_1c_1}$

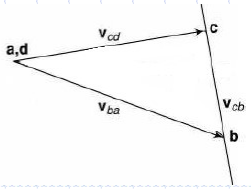
$\mathbf{f^c_{cd} + f^t_{cd} = f^c_{ba} + f^t_{ba} + f^c_{cb} + f^t_{cb}}$

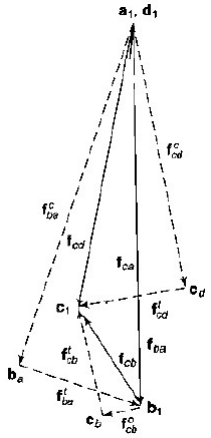
$\mathbf{d_1\,c_d + c_d\,c_1 = a_1\,b_a + b_a\,b_1 + b_1\,c_b + c_b\,c_1}$

SN	Vector	Magnitude	Direction	Sense
1.	$\mathbf{f^c_{ba}}$ or $\mathbf{a_1b_a}$	$\frac{(ab)^2}{AB}$	$\parallel AB$	$\rightarrow A$
2.	$\mathbf{f^t_{ba}}$ or $\mathbf{b_ab_1}$	$\alpha \times AB$	$\perp AB$ or $\mathbf{a_1\,b_a}$ or $\parallel \mathbf{ab}$	$\rightarrow \mathbf{b}$
3.	$\mathbf{f^c_{cb}}$ or $\mathbf{b_1c_b}$	$\frac{(bc)^2}{BC}$	$\parallel BC$	$\rightarrow B$
4.	$\mathbf{f^t_{cb}}$ or $\mathbf{c_bc_1}$	-	$\perp BC$ or $\mathbf{b_1c_b}$	-
5.	$\mathbf{f^c_{cd}}$ or $\mathbf{d_1c_d}$	$\frac{(dc)^2}{DC}$	$\parallel DC$	$\rightarrow D$
6.	$\mathbf{f^t_{cd}}$ or $\mathbf{c_dc_1}$	-	$\perp DC$ or $\mathbf{d_1c_d}$	-

Four Link Mechanism
4R Planar Mechanism







Angular Acceleration of Links

Link *BC*

$\mathbf{f^t_{cb} = c_b\,c_1}$

$f^t_{cb} = \alpha_{cb} CB \quad \therefore \alpha_{cb} = f^t_{cb}/CB \quad (C.C.W)$

Link *CD*

$\mathbf{f^t_{cd} = c_d\,c_1} \quad \therefore \alpha_{cd} = \frac{f^t_{cd}}{CD} = \frac{c_d c_1}{CD}$

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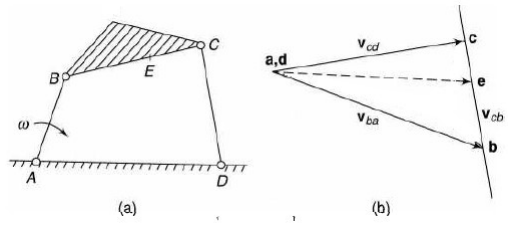
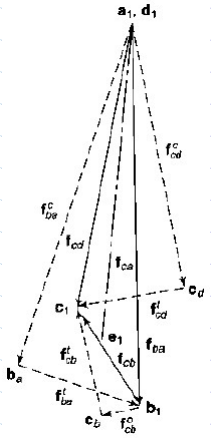
Four Link Mechanism

- **4R Planar Mechanism**

Acceleration of Intermediate Point (Point E on Link BC)





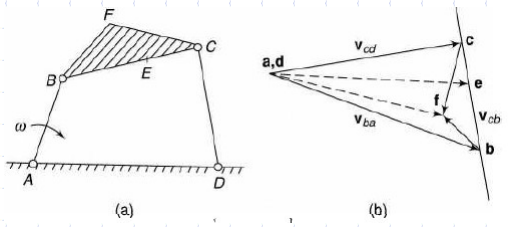
$$\frac{b_1e_1}{b_1c_1} = \frac{BE}{BC}$$

Here, $\overrightarrow{a_1e_1}$: *Total Acceleration of Point E on Link BC*

Four Link Mechanism

- **4R Planar Mechanism**

Acceleration of Offcet Point (Point F on Link BC)



$\Delta b_1c_1f_1$ is similar to ΔBCF.

$\Delta b_1c_1f_1$ is acceleration image.

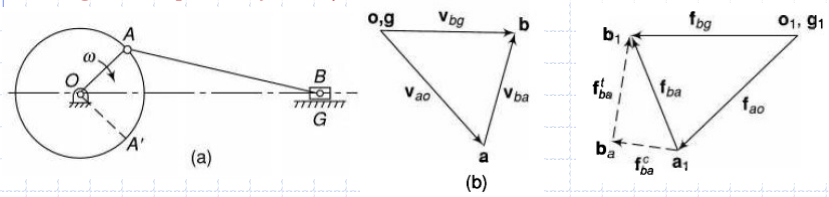
Here, $\overrightarrow{a_1f_1}$: *Total Acceleration of Point F on Link BC*

Four Link Mechanism

- **3R-1P Planar Mechanism (Slider Crank Mechanism)**

OG: Fixed Link, OA: Crank, AB: Coupler, B: Slider

ω: Angular Speed of OA (Clockwise)



$\mathbf{f}_{bo} = \mathbf{f}_{ba} + \mathbf{f}_{ao}$

$\mathbf{f}_{bg} = \mathbf{f}_{ao} + \mathbf{f}_{ba} = \mathbf{f}_{ao} + \mathbf{f}^c_{ba} + \mathbf{f}^t_{ba}$

$\mathbf{g_1\, b_1} = \mathbf{o_1\, a_1} + \mathbf{a_1\, b_a} + \mathbf{b_a\, b_1}$

SN	Vector	Magnitude	Direction	Sense
1.	$\mathbf{f}_{ao}$ or $\mathbf{o_1 a_1}$	$\frac{(oa)^2}{OA}$	$\parallel OA$	$\rightarrow$ O
2.	$\mathbf{f}^c_{ba}$ or $\mathbf{a_1\, b_a}$	$\frac{(ab)^2}{AB}$	$\parallel AB$	$\rightarrow$ A
3.	$\mathbf{f}^t_{ba}$ or $\mathbf{b_a\, b_1}$	-	$\perp AB$	-
4.	$\mathbf{f}_{bg}$ or $\mathbf{g_1 b_1}$	-	$\parallel$ to line of motion of B	-

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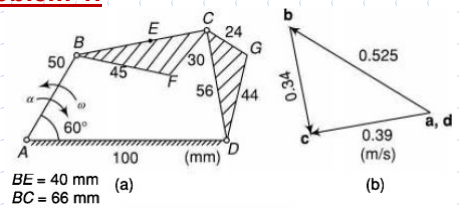
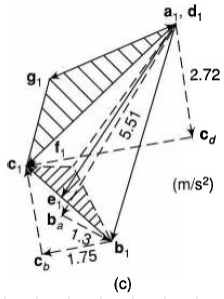
Problem 1.





SN	Vector	Magnitude (m/s²)	Direction	Sense
1.	$\mathbf{f}^c_{ba}$ or $\mathbf{a_1\, b_a}$	$\frac{(ab)^2}{AB} = \frac{(0.525)^2}{0.05} = 5.51$	$\parallel AB$	$\rightarrow A$
2.	$\mathbf{f}^t_{ba}$ or $\mathbf{b_a b_1}$	$\alpha \times AB = 26 \times 0.05 = 1.3$	$\perp AB$ or $\parallel$ **ab**	$\rightarrow$ **a**
3.	$\mathbf{f}^t_{cb}$ or $\mathbf{b_1 c_b}$	$\frac{(bc)^2}{BC} = \frac{(0.34)^2}{0.066} = 1.75$	$\parallel BC$	$\rightarrow B$
4.	$\mathbf{f}^t_{cb}$ or $\mathbf{b_b c_1}$	-	$\perp B$	-
5.	$\mathbf{f}^c_{cd}$ or $\mathbf{d_1 c_d}$	$\frac{(dc)^2}{DC} = \frac{(0.39)^2}{0.56} = 2.72$	$\parallel DC$	$\rightarrow D$
6.	$\mathbf{f}^t_{cd}$ or $\mathbf{c_d c_1}$	-	$\perp B$	-

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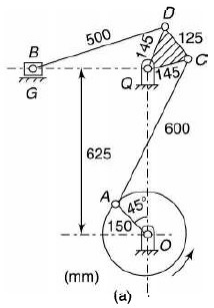
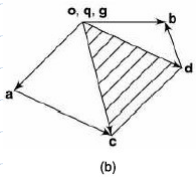
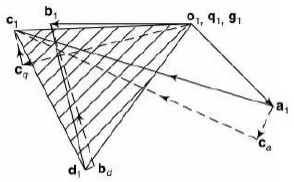
Problem 2.



(a)



(b)



(c)

$\mathbf{f}_{cq} = \mathbf{f}_{ao} + \mathbf{f}_{ca}$

$\mathbf{q_1 c_1} = \mathbf{o_1 a_1} + \mathbf{a_1 c_1}$

$\mathbf{f}^t_{cq} + \mathbf{f}^t_{cq} = \mathbf{f}^t_{ao} + \mathbf{f}^t_{ca} + \mathbf{f}^t_{ca}$

$\mathbf{q_1 c_q} + \mathbf{c_q c_1} = \mathbf{o_1 a_1} + \mathbf{a_1 c_a} + \mathbf{c_a a_1}$

Draw $\Delta \mathbf{c_1 q_1 d_1}$ similar to ΔCQD

$\mathbf{f}_{bg} = \mathbf{f}_{dq} + \mathbf{f}_{bd}$

$\mathbf{g_1 b_1} = \mathbf{q_1 d_1} + \mathbf{d_1 b_1}$

$\mathbf{f}_{bg} = \mathbf{f}_{dq} + \mathbf{f}^c_{bd} + \mathbf{f}^t_{bd}$

$\mathbf{g_1 b_1} = \mathbf{q_1 d_1} + \mathbf{d_1 b_d} + \mathbf{b_d b_1}$

SN	Vector	Magnitude (m/s²)	Direction	Sense
1.	$\mathbf{f_{ao}}$ or $\mathbf{o_1 a_1}$	$\frac{(oa)^2}{OA} = \frac{(0.94)^2}{0.15} = 5.92$	$\|\| OA$	$\rightarrow O$
2.	$\mathbf{f^c_{ca}}$ or $\mathbf{a_1 c_a}$	$\frac{(ac)^2}{AC} = \frac{(1.035)^2}{0.60} = 1.79$	$\|\| AC$	$\rightarrow A$
3.	$\mathbf{f^t_{ca}}$ or $\mathbf{c_a c_1}$	-	$\perp AC$	-
4.	$\mathbf{f^c_{cq}}$ or $\mathbf{q_1 c_q}$	$\frac{(qc)^2}{QC} = \frac{(1.14)^2}{0.145} = 8.96$	$\|\| QC$	$\rightarrow Q$
5.	$\mathbf{f^t_{cq}}$ or $\mathbf{c_q c_1}$	-	$\perp QC$	-

SN	Vector	Magnitude (m/s²)	Direction	Sense
1.	$\mathbf{f}_{dq}$ or $\mathbf{q_1 d_1}$	Already drawn	-	-
2.	$\mathbf{f^c_{bd}}$ or $\mathbf{d_1 b_d}$	$\frac{(db)^2}{DB} = 0.49$	$\|\| DB$	$\rightarrow D$
3.	$\mathbf{f^t_{bd}}$ or $\mathbf{b_d b_1}$	-	$\perp DB$	-
4.	$\mathbf{f}_{bg}$ or $\mathbf{g_1 b_1}$	-	|| to slider motion	-

$\alpha_{cqd} = \frac{\mathbf{f^t_{cq}} \text{ or } \mathbf{c_q c_1}}{QC}$ $\quad$ $\alpha_{bd} = \frac{\mathbf{f^t_{bd}} \text{ or } \mathbf{b_d b_1}}{BD}$

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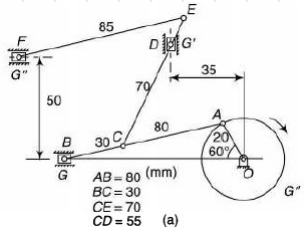
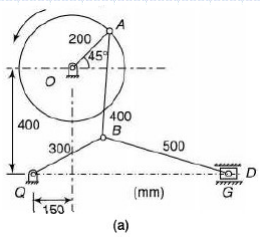
Problem 3.



(a)

Problem 4.



(a)

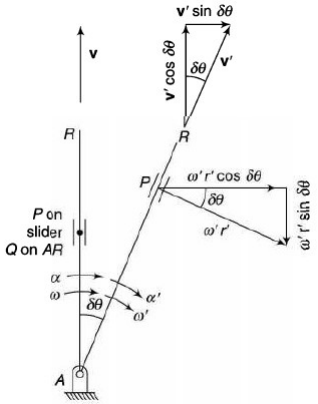
12

Crank and Slotted Lever Mechanism

Coriolis Acceleration Component

ω = angular velocity of the link
α = angular acceleration of the link
v = linear velocity of the slider on the link
f = linear acceleration of the slider on the link
r = radial distance of point P on the slider

$\omega' = \omega + \alpha\delta t$ = angular velocity of the link
$v' = v + f.\delta t$ = linear velocity of the slider on the link
$r' = r + \delta r$ = radial distance of the slider



Acceleration of P Parallel to AR

Initial velocity of P along $AR = v = v_{pq}$
Final velocity of P along $AR = v'\cos\delta\theta - \omega' r' \sin\delta\theta$
Change of velocity along $AR = (v'\cos\delta\theta - \omega' r' \sin\delta\theta) - v$
Acceleration of P along AR

$$= \frac{(v + f\delta t)\cos\delta\theta - (\omega + \alpha\delta t)(r + \delta r)\sin\delta\theta - v}{\delta t}$$

In the limit, as $\delta t \to 0$
$\cos\delta\theta \to 1$ and $\sin\delta\theta \to \delta\theta$

$$\text{Acceleration of } P \text{ along } AR = f - \omega r\frac{d\theta}{dt}$$
$$= f - \omega r w = f - \omega^2 r$$
$$= \text{Acc. of slider–centripetal. acc.}$$

Crank and Slotted Lever Mechanism

Coriolis Acceleration Component

Acceleration of P Perpendicular to AR

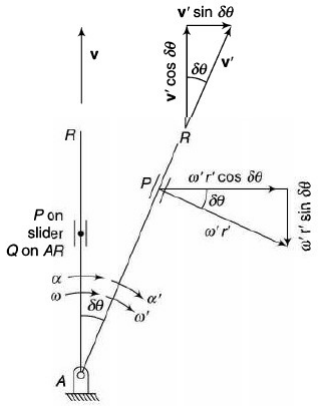
Initial velocity of $P \perp$ to $AR = \omega r$
Final velocity of $P \perp$ to $AR = v' \sin\delta\theta + \omega' r' \cos\delta\theta$
Change of velocity $\perp$ to $AR = (v'\sin\delta\theta + \omega' r'\cos\delta\theta) - \omega r$
Acceleration of $P \perp$ to AR

$$= \frac{(v + f\delta t)\cos\delta\theta - (\omega + \alpha\delta t)(r + \delta r)\cos\delta\theta - \omega r}{\delta t}$$

In the limit, as $\delta t \to 0$

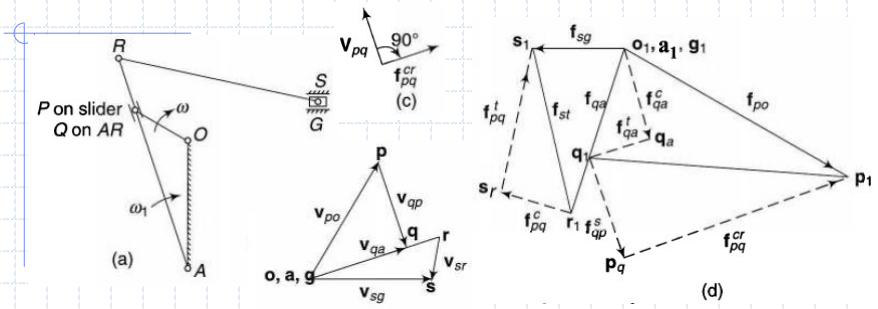
$$\cos\delta\theta \to 1 \quad \text{and} \quad \sin\delta\theta \to \delta\theta$$

$$\text{Acceleration of } P \perp \text{ to } AR = v\frac{d\theta}{dt} + \omega\frac{dr}{dt} + ra$$
$$= v\omega + \omega v + r\alpha = 2\omega v + r\alpha$$
$$= 2\omega v + \text{tangential acc.}$$



The component $2\omega.v$ is known as the *Coriolis acceleration*

Crank and Slotted Lever Mechanism



$\mathbf{f}_{po} = \mathbf{f}_{qa} + \mathbf{f}_{pq}$

$= f^c_{qa} + f^t_{qa} + f^s_{pq} + f^{cr}_{pq}$

$\mathbf{o_1\, p_1} = \mathbf{a_1\, q_a} + \mathbf{q_a\, q_1} + \mathbf{q_1\, p_q} + \mathbf{p_q\, p_1}$

SN	Vector	Magnitude	Direction	Sense
1.	$\mathbf{f}_{po}$ or $\mathbf{o_1\, p_1}$	$\omega^2 \times OP$	$\parallel OP$	$\rightarrow O$
2.	$\mathbf{f}^c_{qa}$ or $\mathbf{a_1\, q_a}$	$\frac{(\mathbf{aq})^2}{AQ}$	$\parallel AQ$	$\rightarrow A$
3.	$\mathbf{f}^t_{qa}$ or $\mathbf{q_a\, q_1}$	-	$\perp AQ$ or $\mathbf{a_1\, q_a}$	-
4.	$\mathbf{f}^s_{pq}$ or $\mathbf{q_1\, p_q}$	-	$\parallel AR$	-
5.	$\mathbf{f}^{cr}_{pq}$ or $\mathbf{p_q\, p_1}$	Coriolis component*	$\perp AR$	Refer*

* $f^{cr}_{pq} = 2\omega_1 v_{pq}$

$= 2\left(\frac{\mathbf{aq}}{AQ}\right) qp$

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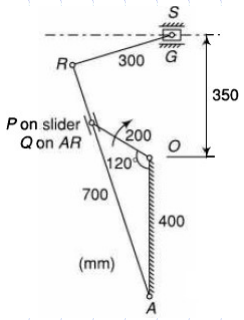
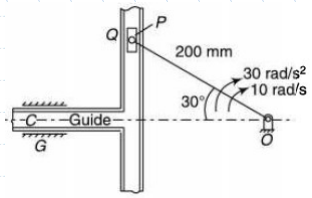
Crank and Slotted Lever Mechanism

Problem 1.



Problem 2.



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