MEC2120
Kinematics of Machines

Unit 1

Mechanics

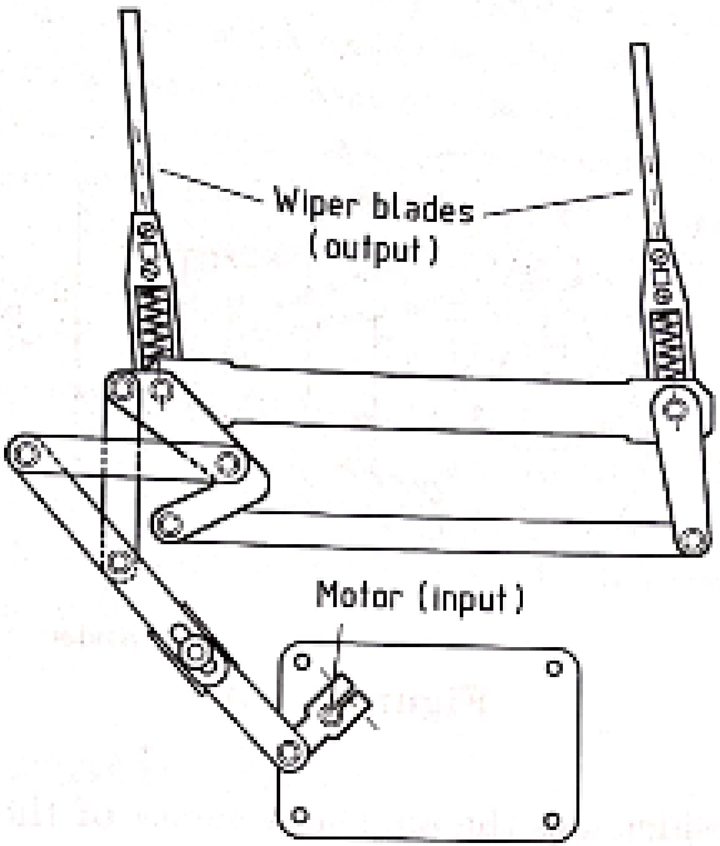
- Science which describes and predicts the condition of rest or motion of bodies under the action of forces
  - Mechanics of Rigid Bodies
  - Mechanics of Deformable Bodies
  - Mechanics of Fluids

Mechanics of Rigid Bodies

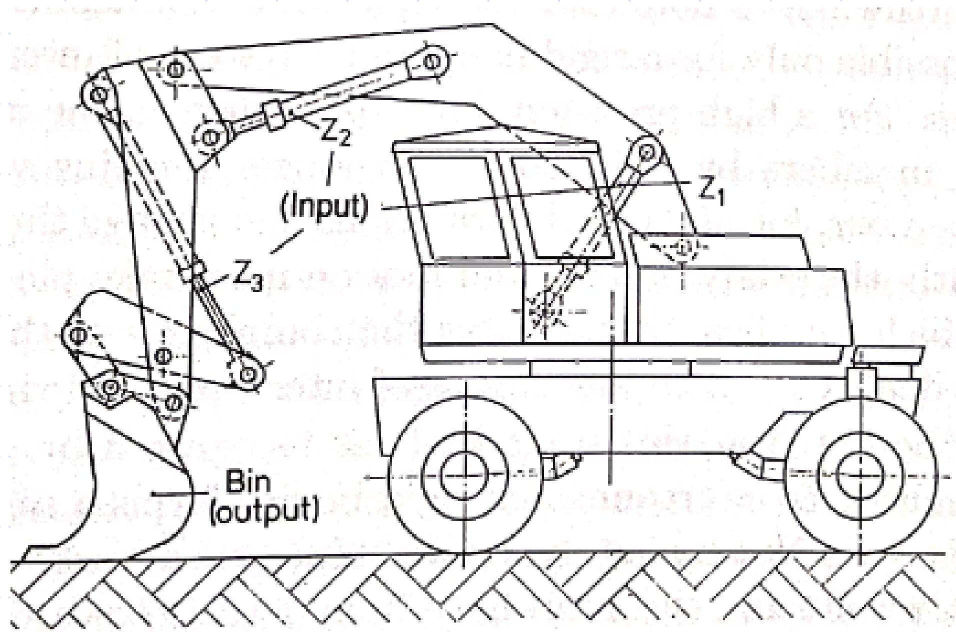
- ✓ Statics: dealing with bodies at rest.
- ✓ Dynamics : dealing with bodies in motion.
  - ❖ Kinematics
    - It describes the motion of objects
    - It is the study of description of motion
  - ❖ Kinetics
    - studies forces that cause changes of motion
    - It is the study of explanation of motion

Mechanisms and Machines

- Mechanisms and Machines refer to devices which transfer mechanical motions and forces from a source to an output member (IFToMM).
- If the idea of transferring motion predominates→ Mechanism
- when substantial forces are also involved → Machine



Wiper Mechanism



Dumping Machine

Mechanisms and Machines

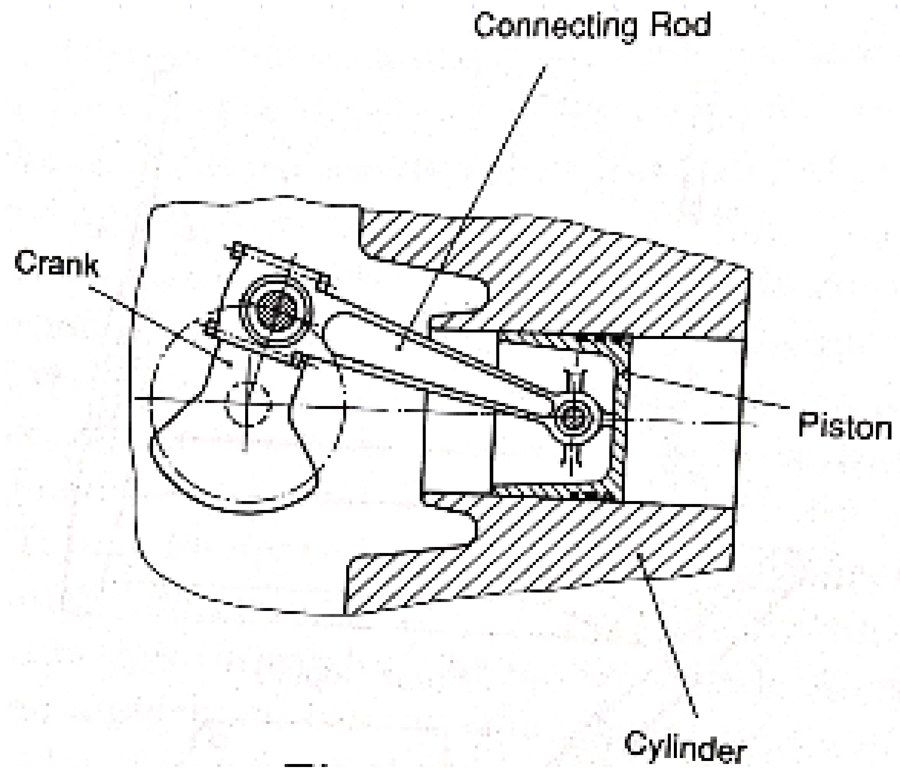
Kinematics

Study of motion of Connected rigid bodies

Kinetics

For the same system,

- the word mechanism is used if studying kinematics
- the word machine is used if studying kinetics



Mechanisms and Machines

Kinematics → Kinematic Analysis

Kinematics → Kinematic Synthesis

- In Kinematic analysis, motion characteristics such as displacement, velocity, acceleration are investigated for given geometric parameters.
- Kinematic synthesis deals with the inverse problem.

Types of Constrained Motions

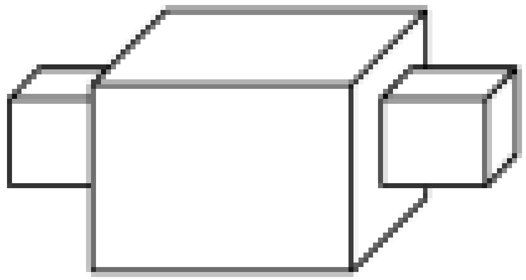
Completely constrained motion.

- When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.

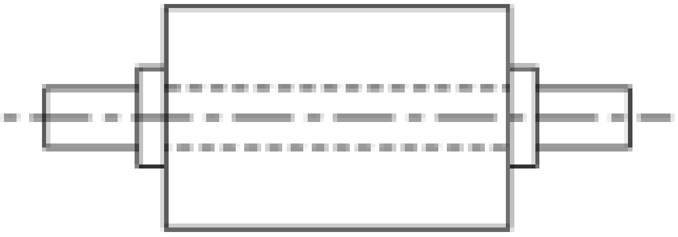
e.g.

a) The motion of a square bar in a square hole.

b) The motion of a shaft with collars at each end in a circular hole.



(a)



(b)

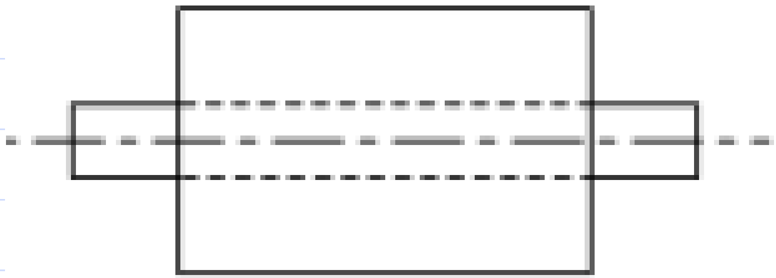
Types of Constrained Motions

Incompletely constrained motion.

- When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion.
- The change in the direction of impressed force may alter the direction of relative motion between the pair.

e.g.

a) A circular bar or shaft in a circular hole, as shown in Fig., is an example of an incompletely constrained motion as it may either rotate or slide in a hole.

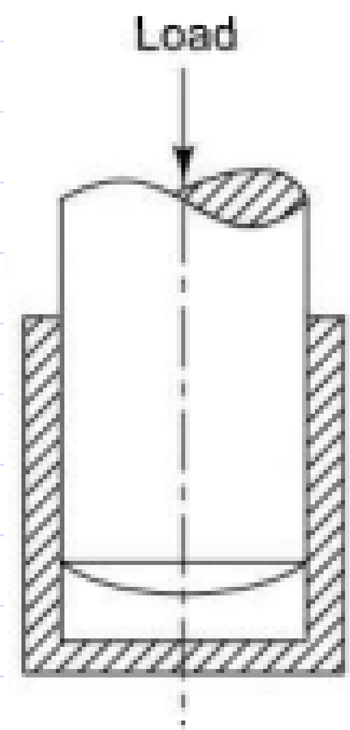


Types of Constrained Motions

Successfully constrained motion.

- When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion.
- e.g.

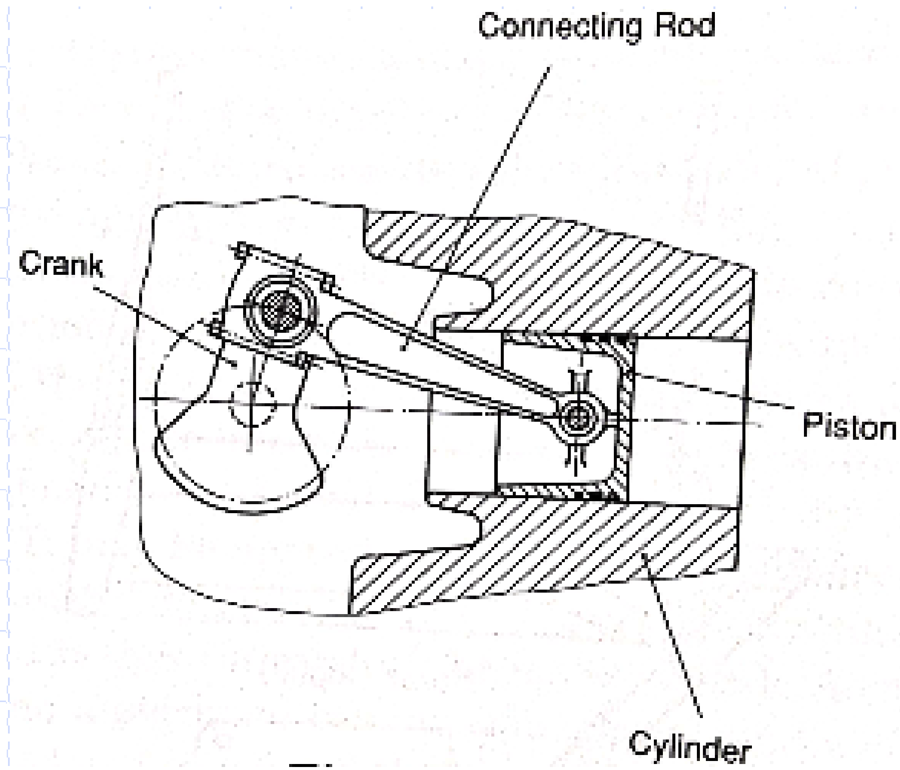
a) A shaft in a foot-step bearing
b) piston reciprocating inside an engine cylinder
c) An I.C. engine valve (these are kept on their seat by a spring)



Footstep bearing

Basic Definitions & Nomenclature

Kinematic Link: A resistant body or a group of resistant bodies with rigid connections preventing their relative movement is called a link or kinematic link.



Slider Crank Mechanism

4 links namely

- Frame and guide
- Crank
- Connecting rod
- Slider (piston)

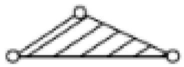
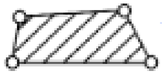
Singular Link

A link which is connected to only one other link

Binary Link

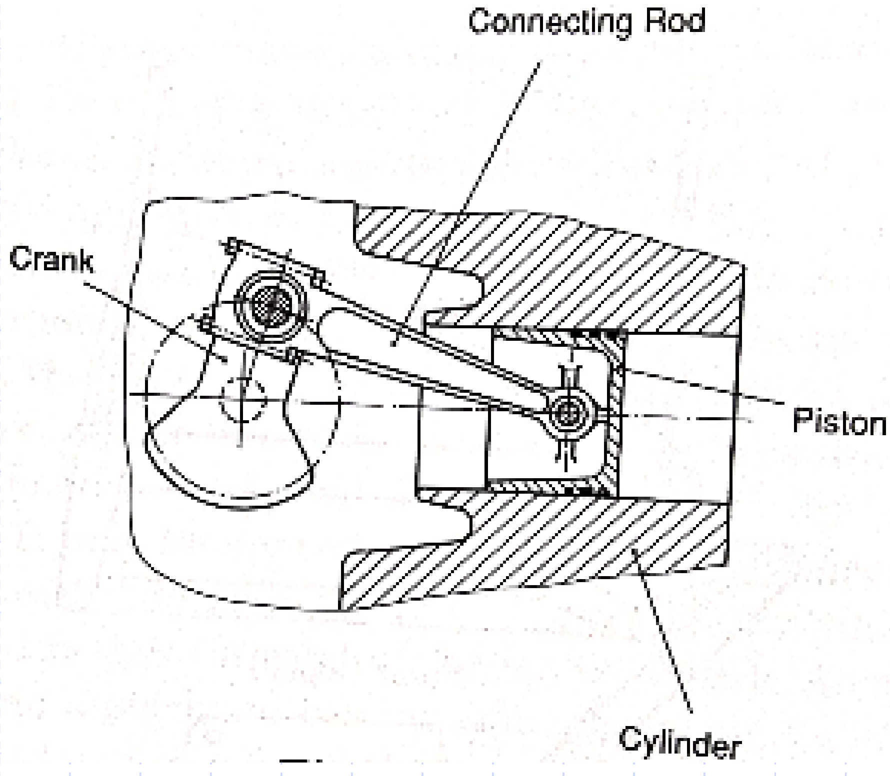
A link which is connected to two other links. 

Ternary Link

A link which is connected to three other links  

Basic Definitions & Nomenclature

Kinematic Pairs: A kinematic pair or simply a pair is a joint of two links having relative motion between them



Slider Crank Mechanism

4 kinematic pairs

- Crank and Frame
- Crank and Connecting rod
- Connecting rod and Slider (piston)
- Slider (piston) and Frame

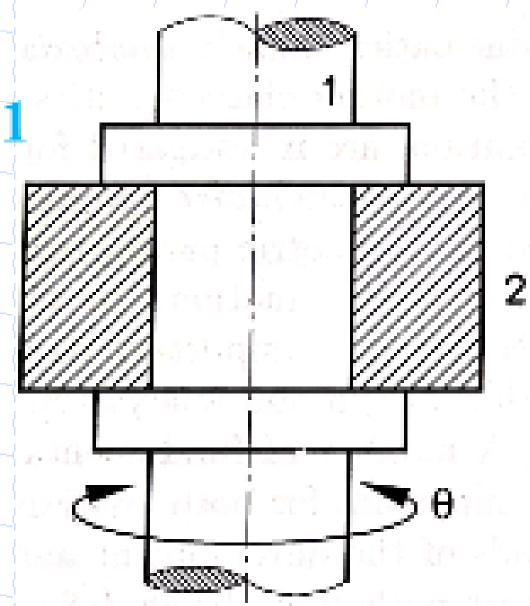
Basic Definitions & Nomenclature

Degrees of Freedom of a kinematic Pair: The number of independent coordinates (pair variables) required to completely specify the relative movement.

Types of Kinematic Pairs
(based on the possible relative movements)

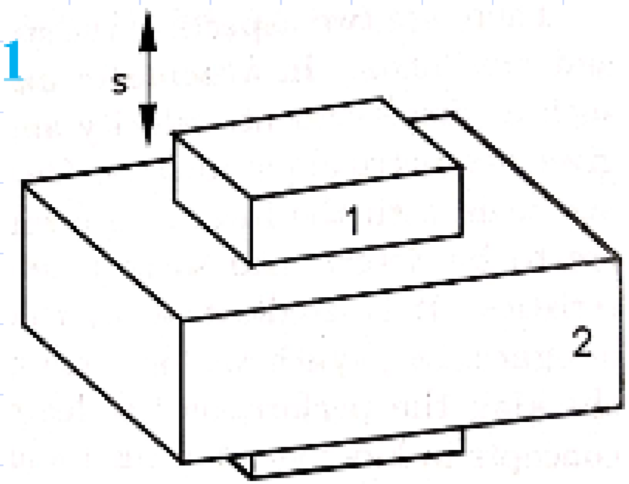
1) Revolute/Turning Pair

D.O.F = 1
P.V: θ



2) Prismatic/Sliding Pair

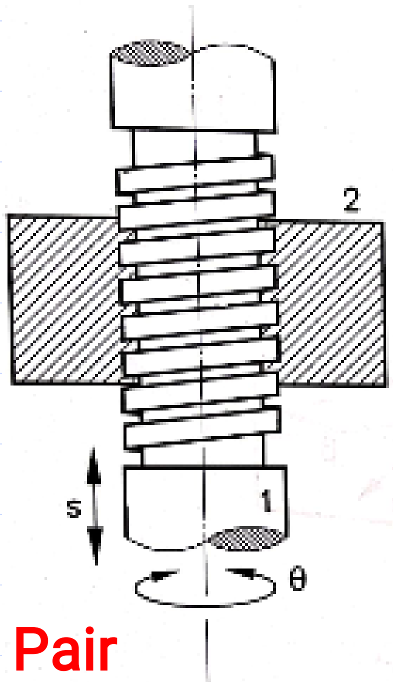
D.O.F = 1
P.V: s



Basic Definitions & Nomenclature

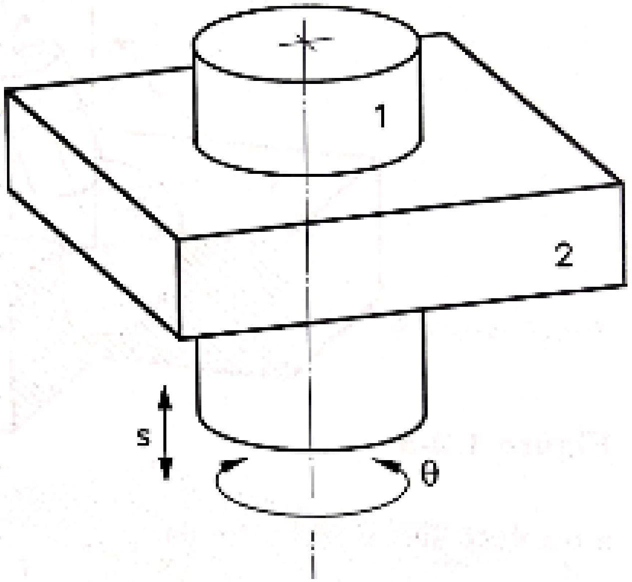
3) Screw/Helical Pair

D.O.F = 1
P.V: θ or s



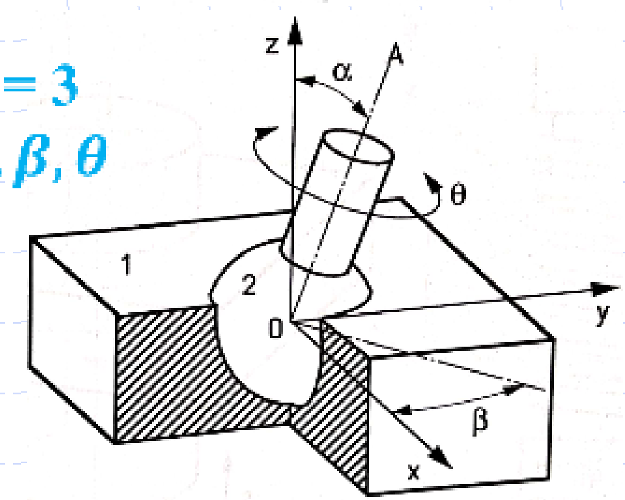
4) Cylindric Pair

D.O.F = 2
P.V: θ & s



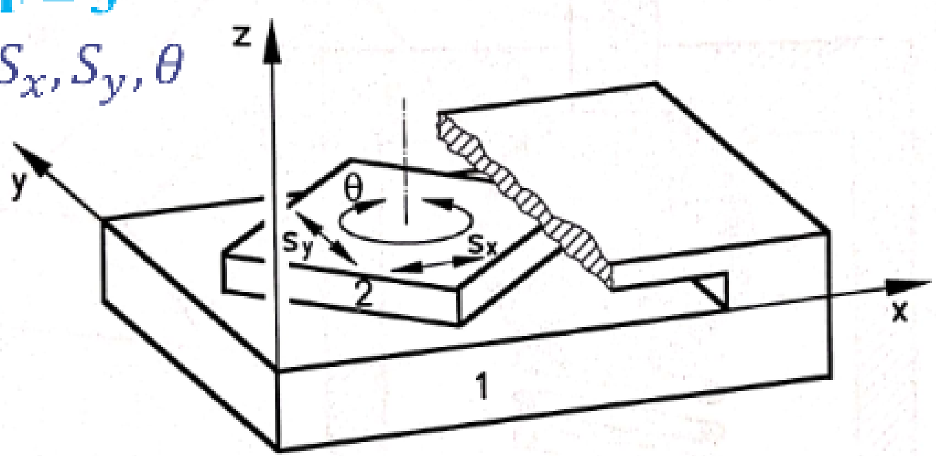
5) Spheric Pair

D.O.F = 3
P.V: α, β, θ



6) Planar Pair

D.O.F = 3
P.V: S_x, S_y, θ

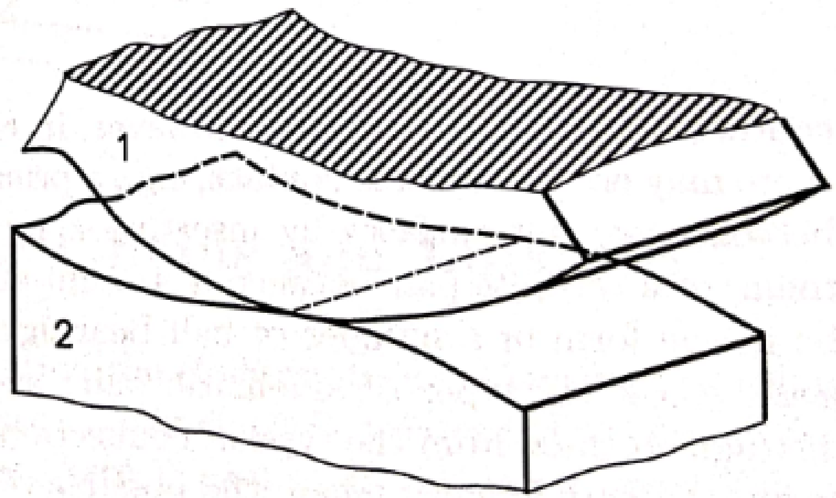


Basic Definitions & Nomenclature

Types of Kinematic Pairs
(based on the nature of contact)

Lower pairs: A pair of links having surface or area contact.
e.g.: Revolute Pair, Prismatic Pair, Screw Pair, Cylindric Pair, Spheric Pair & Planar Pair

Higher pairs: A pair of links having line or point contact.
e.g.: wheel rolling on a surface, cam and follower pair, toothed gears



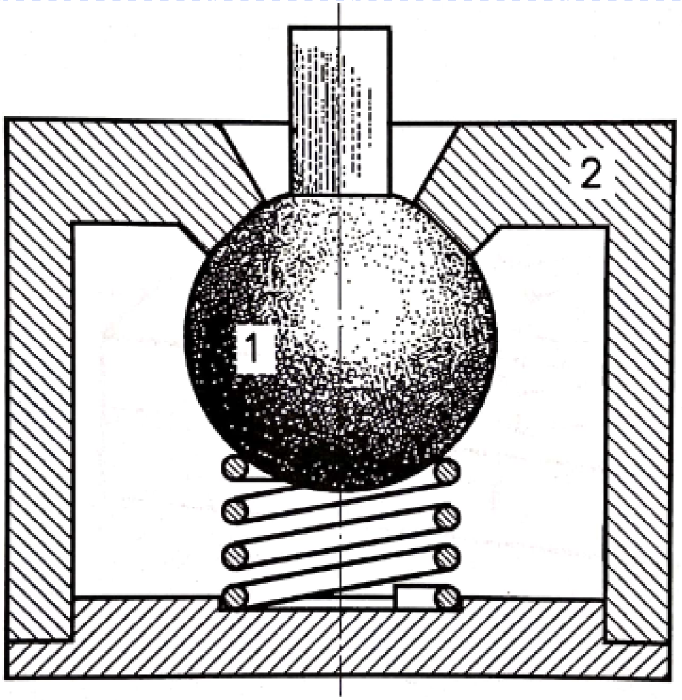
Meshing gear tooths

Basic Definitions & Nomenclature

Types of Kinematic Pairs
(based on the nature of constraint)

Form closed pair: when the contact between the elements of a kinematic pair is maintained only by the geometric forms of the contacting surfaces.
e.g.: all the lower pairs

Force closed pair: when the contact between the elements of a kinematic pair is maintained by an external force (e.g. that of spring)
.
e.g.:

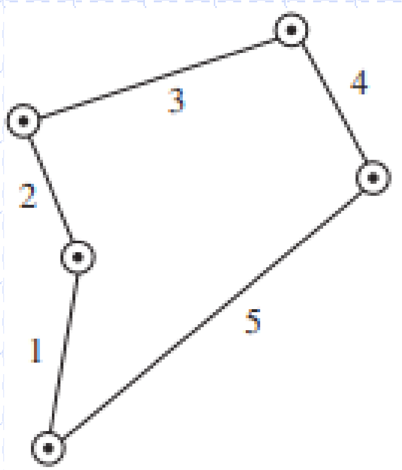


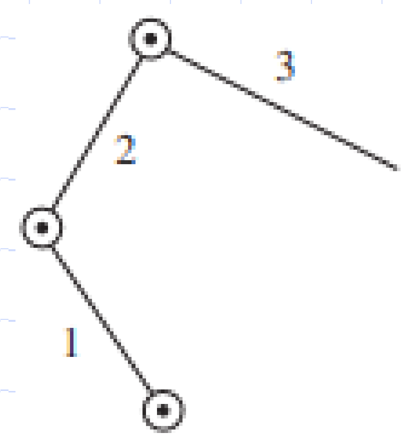
Basic Definitions & Nomenclature

Kinematic Chain : A series of link connected by kinematic pairs.

Kinematic Chain →

- Closed Chain
  The connected links form a closed loop.
- Open Chain
  The connected links form an open loop.



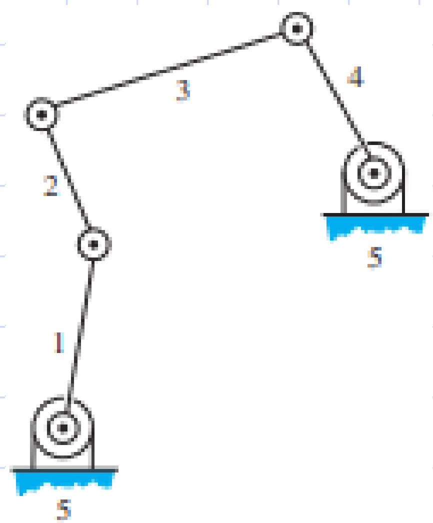


Kinematic Chain →

- Simple Chain
- Compound Chain

Basic Definitions & Nomenclature

- **A mechanism** can be defined as a closed kinematic chain with one fixed link.



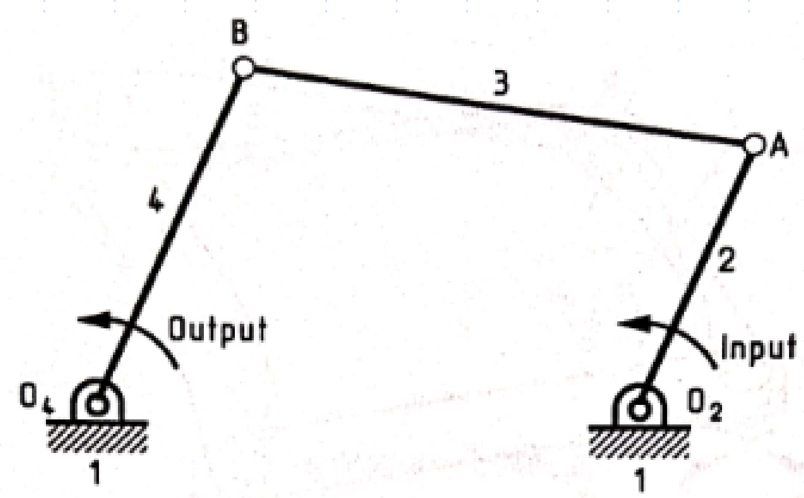
D.O.F. of Mechanism: No. of independent pair variables needed to completely define the relative movements between all its links.
Constrained Mechanism: no. of input motions = D.O.F. of Mechanism

Linkage: A mechanism consisting of only the lower pairs.

Basic Definitions & Nomenclature

Planar Mechanism/linkage: All the points of mechanism move in parallel planes.

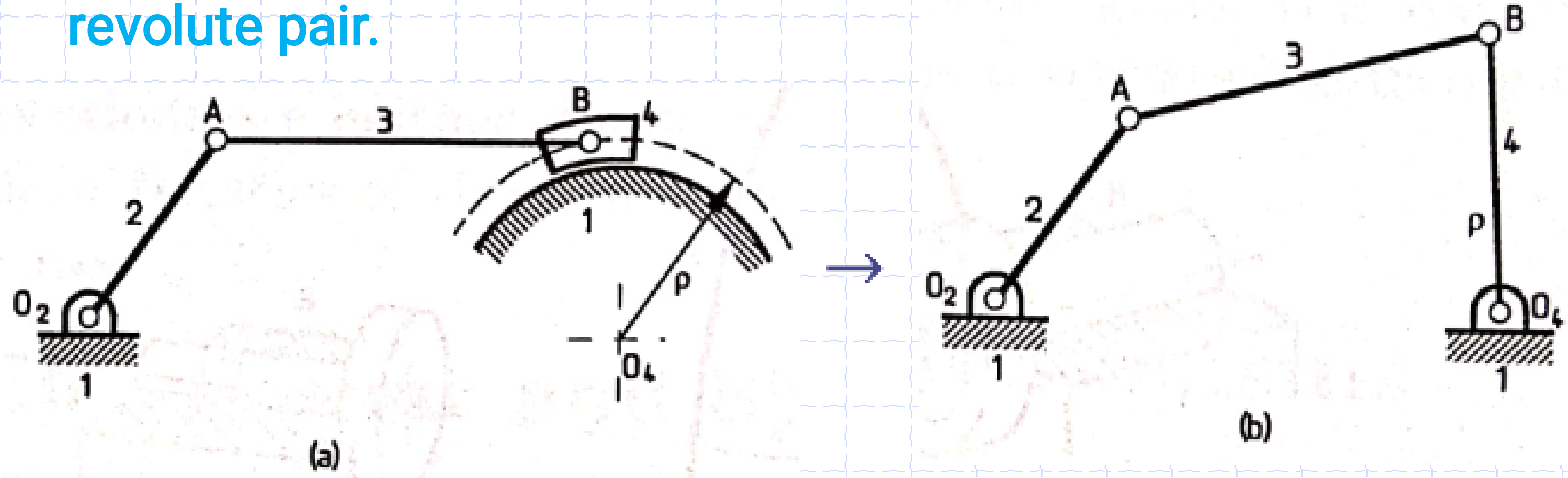
- A planar linkage can have only revolute and prismatic pairs.
- All the revolute axes are normal to the plane of motion.
- All the prismatic pair axes are parallel to the plane.
- A single view perpendicular to the plane of motion reveals the true motions.



Limit and Disguise of Revolute Pair

Revolute and Prismatic pair are the basic building blocks o all lower pairs.

- A prismatic pair can always be thought of as the limit of a revolute pair.



If $\rho \to \infty$, the pair variable transforms from angular movement to linear displacement

If $\rho \to \infty$, $\Rightarrow$ the connection between links 1 and 4 becomes a prismatic pair.

Kutzbach Equation

Planar Mechanism

Let

$n = No.\ of\ links$

$j = No.\ of\ simple\ R\ pairs$

No. of D.O.F of an unconnected rigid body in plane motion = 3

i.e. 2 translational & one rotational D.O.F.

No. of D.O.F of $(n-1)$ unconnected rigid body in plane motion $= 3(n-1)$

Once two links are connected by an R pair,

No of D.O.F. lost = 2

No of D.O.F. left = 1

∴ No. of D.O.F of the mechanism,

$$F = 3(n-1) - 2j$$

This equation is known as **Kutzbach Equation.**

Grubler criterion

Planar Mechanism

If

$\boldsymbol{F = 1}$, the mechanism is a single D.O.F mechanism

$\boldsymbol{F = 2}$, the mechanism is a two D.O.F mechanism

$\boldsymbol{F = 0}$, the assembly is a structure

$\boldsymbol{F = -1}$ or less, the assembly is a statically indeterminate structure

For a single D.O.F mechanism, putting F=1 in the Kutzbach Eq., we get

$$2j - 3n + 4 + 0$$

This simple estimate of constrained movement is known as **Grubler criterion.**

Kutzbach Equation

Planar Mechanism

Kutzbach Equation

$$F = 3(n-1) - 2j$$

Since a Prismatic pair is a special case of a Revolute pair

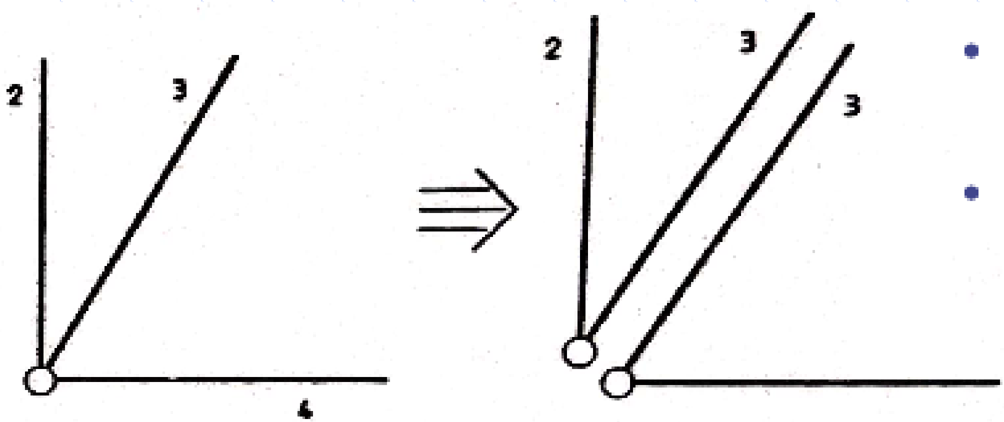
$\therefore$ j include prismatic pairs.

For higher order Pairs:

$$j = j_1 + 2j_2 + 3j_3 + \cdots + ij_i$$

Where

j_i: no of R pairs, each of which connects (i+1) links



- j_2 is equivalent to 2 simple R pairs

  Similarly,
- j_i is equivalent to i nos. of simple R pairs

Kutzbach Equation

Planar Mechanism

Kutzbach Equation for mechanism with higher pair

- A higher pair curtails 1 D.O.F

∴ D.O.F of mechanism having h nos. of higher pairs can be written as

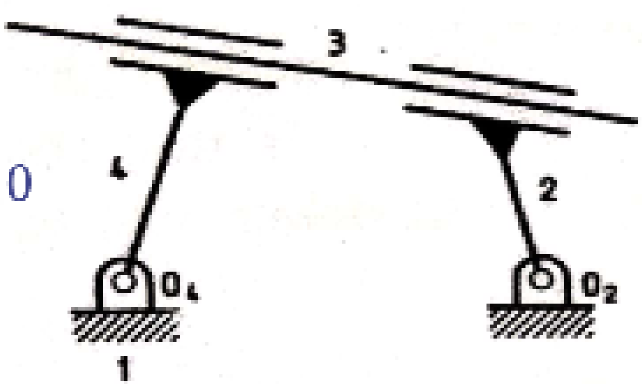
$$F = 3(n-1) - 2j - h$$

Effective D.O.F of mechanism with redundant degrees of freedom

$$F_e = 3(n-1) - 2j - h - F_r$$

$F_e = 0$

$F_r = 1$



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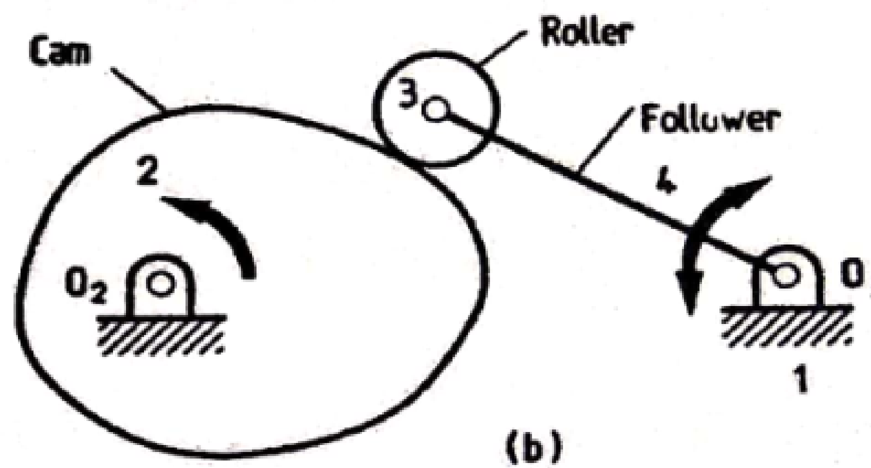
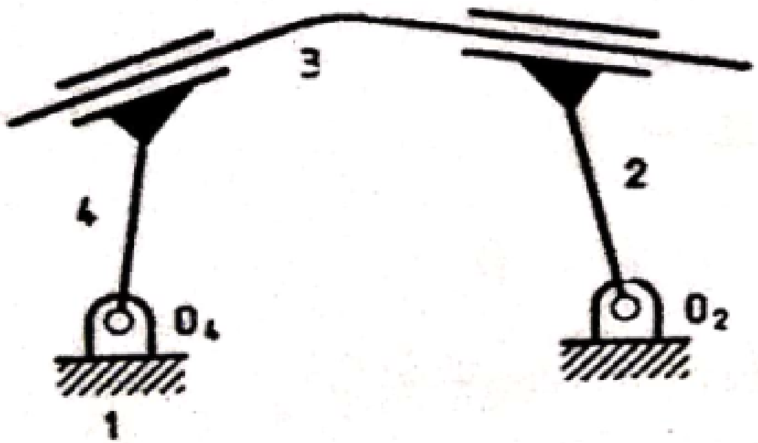


(b)

$F_e = 1$

$F_r = 1$

Kutzbach Equation



$F_e = 0$

$F_r = 1$

Kinematic Inversions

- Mechanisms that are derived from the same kinematic chain but have a different link fixed to ground.
- The relative motions of the links are the same in kinematic inversions (i.e., the motions at the joints are the same), but the absolute motions of the links are different, since they are being referenced to different links.

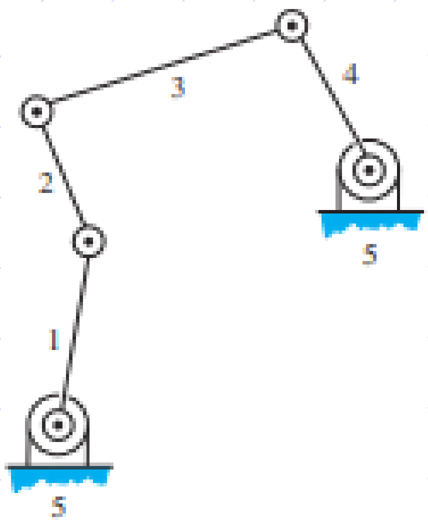


Figure (a)

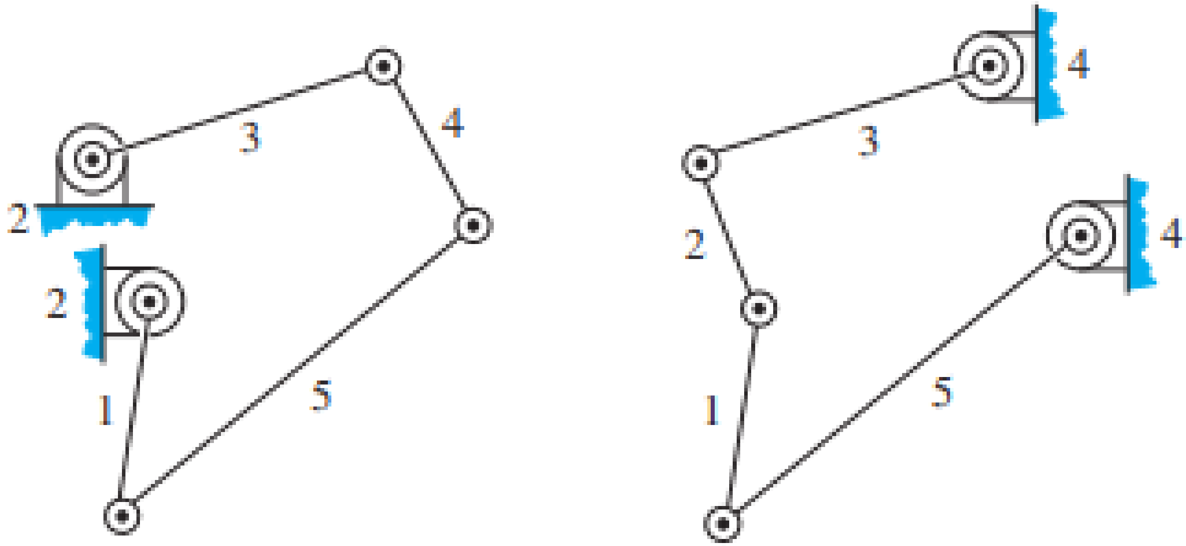


Figure (b): Two of the four possible kinematic inversions of the mechanism in Figure (a)

Kinematic Inversions

Inversions Of 4 Bar Kinematic Chain

- If in a four bar kinematic chain all links are free, motion will be unconstrained.
- From a four link kinematic chain, four different mechanisms can be obtained by fixing each of the four links turn by turn.
- All these mechanisms are called inversions of the parent kinematic chain.
- By this principle of inversions of a four link chain, several useful mechanisms can be obtained.

Kinematic Inversions

Inversions of Kinematic Chain with all the four kinematic pairs as revolute pairs

- All the four inversions of such a chain are identical.
- However, by suitably altering the proportions of lengths of links 1, 2, 3 and 4 respectively several mechanisms are obtained.

Crank-rocker Mechanism

$$(l_1 + l_2) < (l_3 + l_4)$$

$$(l_2 + l_3) < (l_1 + l_4)$$

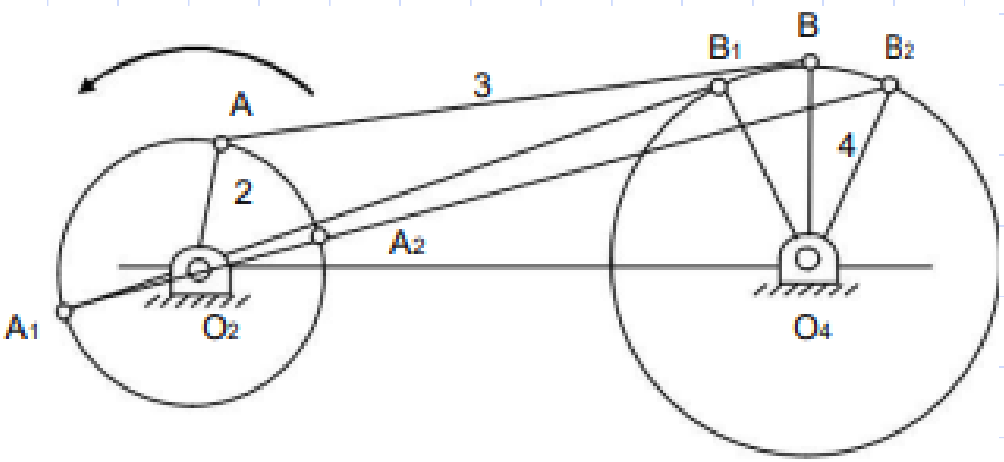
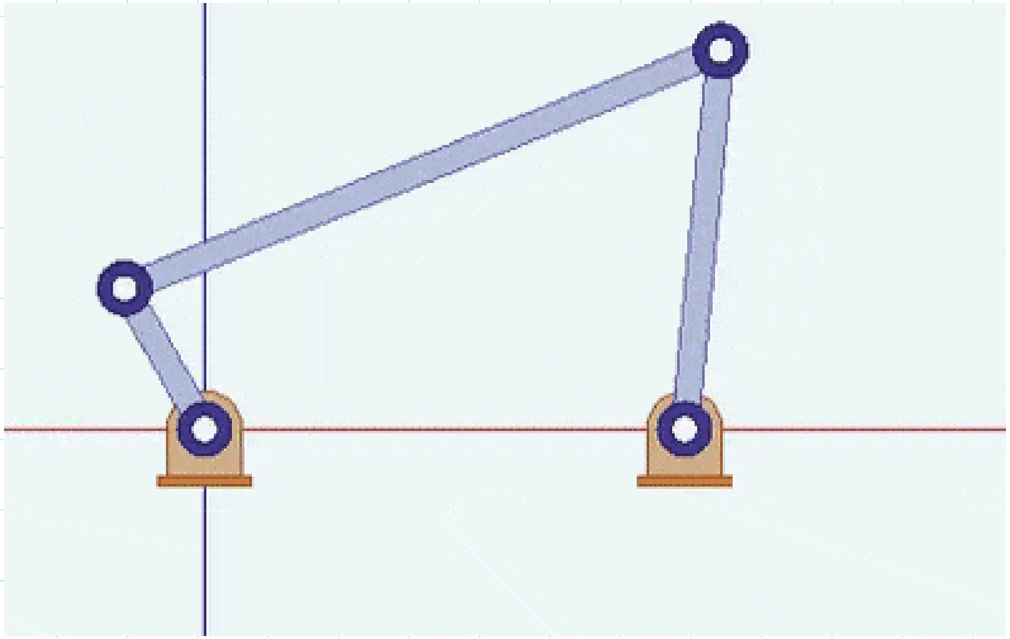


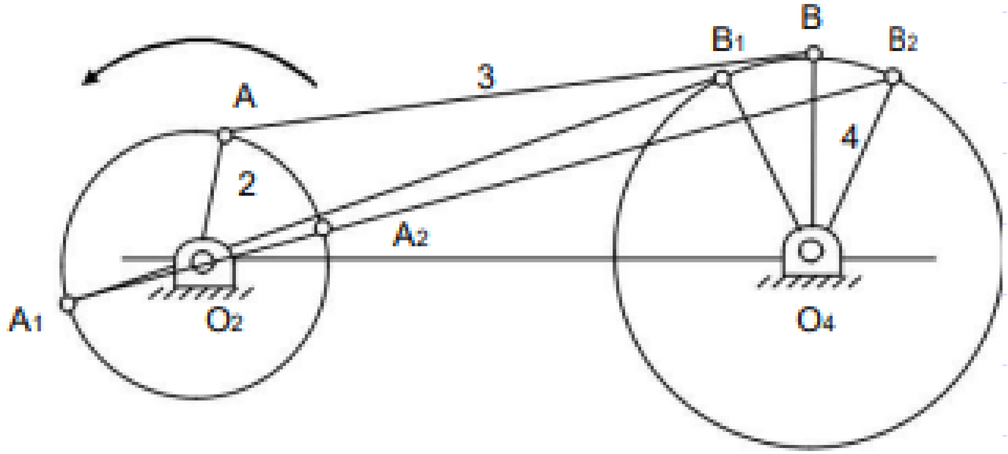
Figure: Crank-rocker Mechanism

Kinematic Inversions

Inversions of Kinematic Chain with all the four kinematic pairs as revolute pairs

Crank-rocker Mechanism





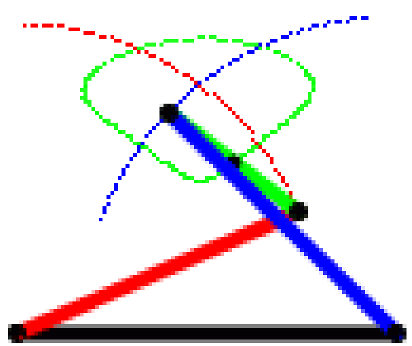
$$(l_1 + l_2) < (l_3 + l_4)$$

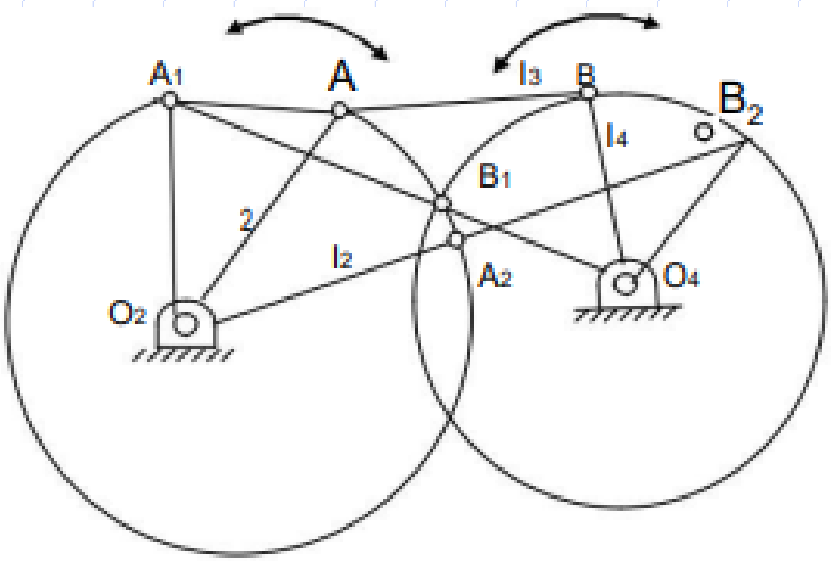
$$(l_2 + l_3) < (l_1 + l_4)$$

Kinematic Inversions

Inversions of Kinematic Chain with all the four kinematic pairs as revolute pairs

Double Lever Mechanism or Rocker-Rocker Mechanism





$$(l_3 + l_4) < (l_1 + l_2)$$

$$(l_2 + l_3) < (l_1 + l_4)$$

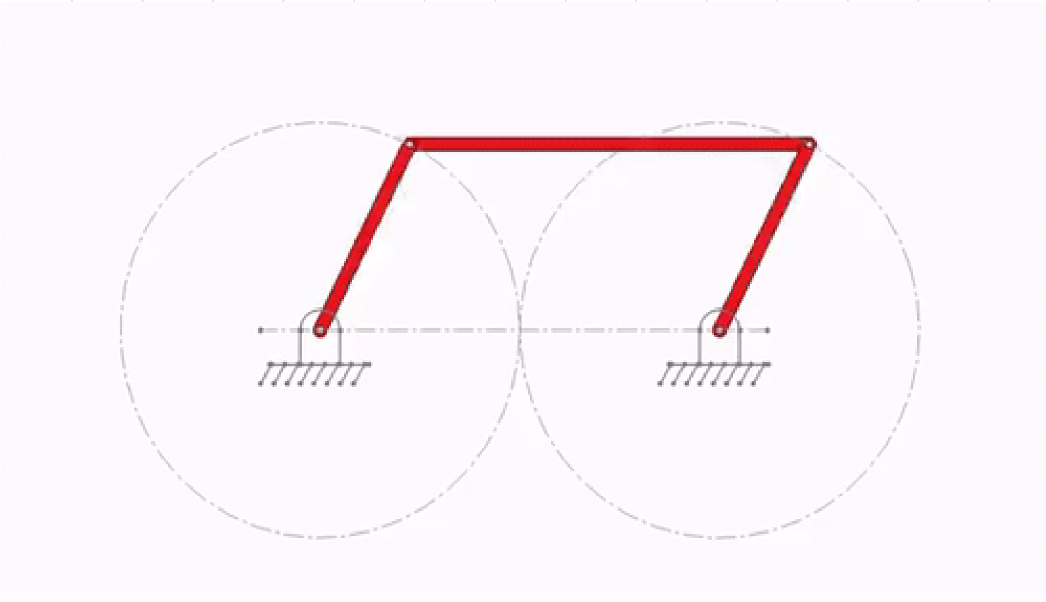
Kinematic Inversions

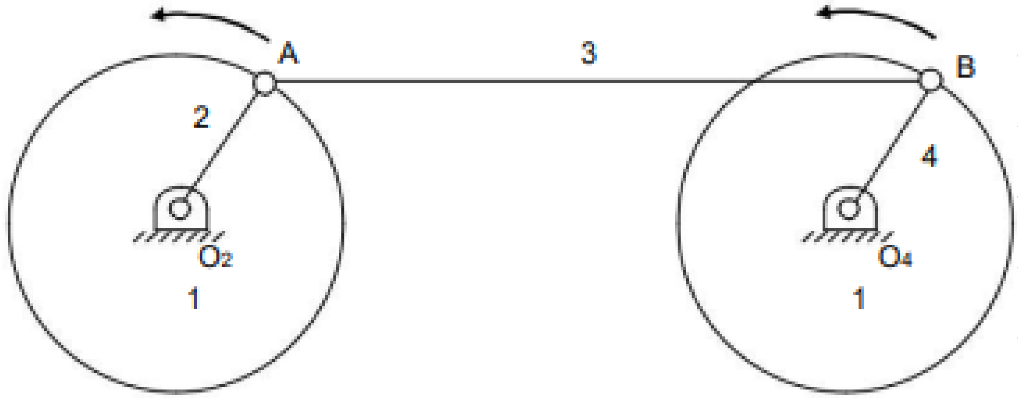
Inversions of Kinematic Chain with all the four kinematic pairs as revolute pairs

Double Crank Mechanism

- The links 2 and 4 of the double crank mechanism make complete revolutions. There are two forms of this mechanism.

  - Parallel Crank Mechanism



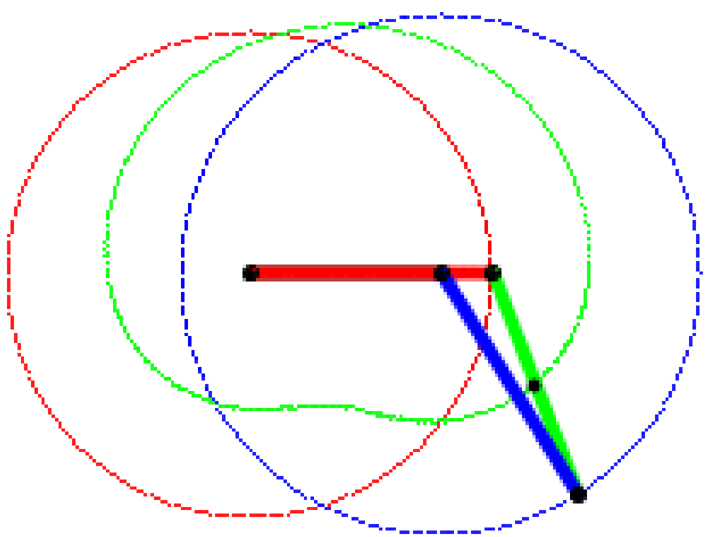


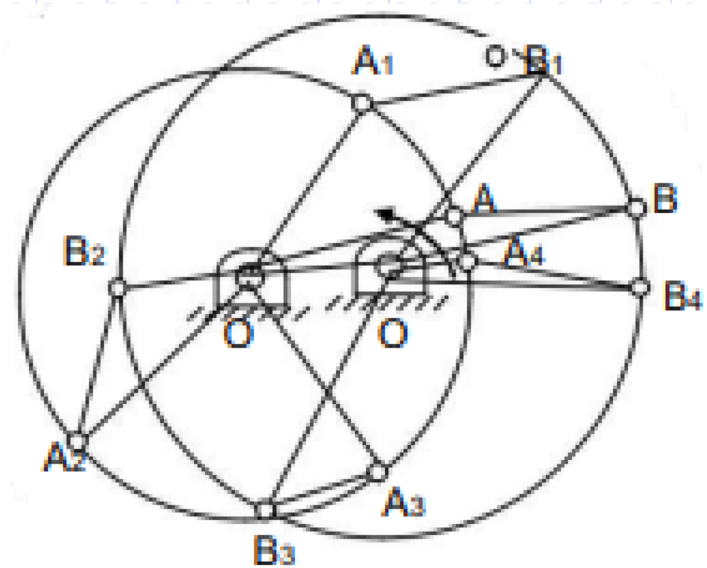
Kinematic Inversions

Inversions of Kinematic Chain with all the four kinematic pairs as revolute pairs

Double Crank Mechanism

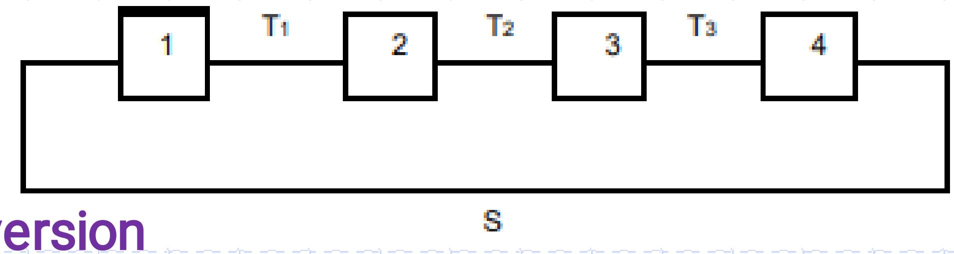
- Drag Link Mechanism



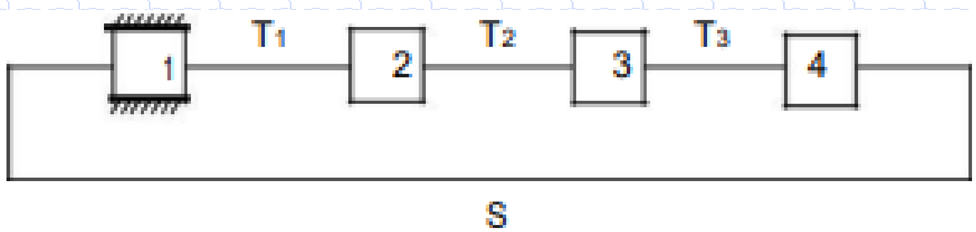


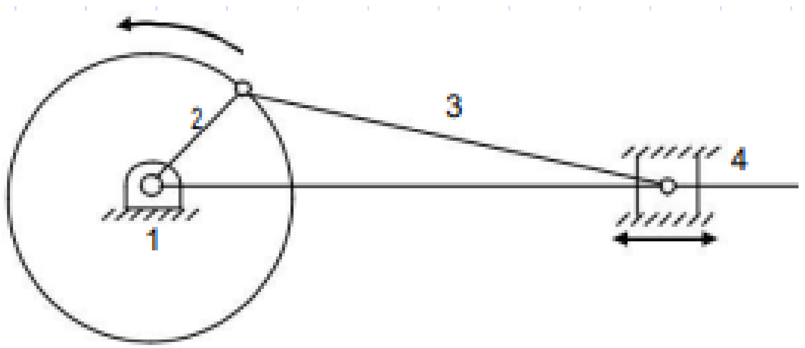
Kinematic Inversions

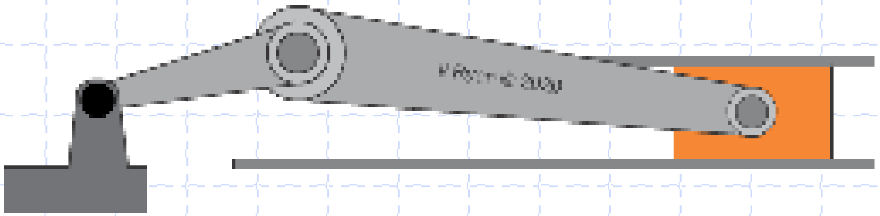
Inversions of Kinematic Chain with three revolute pairs and one sliding pair



First Inversion

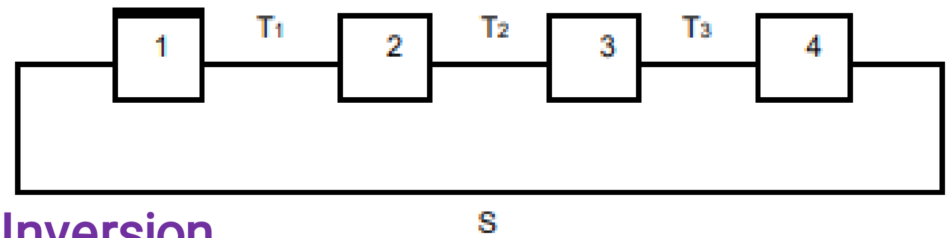




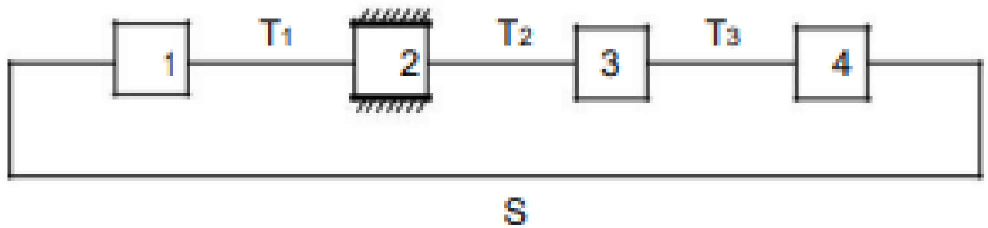


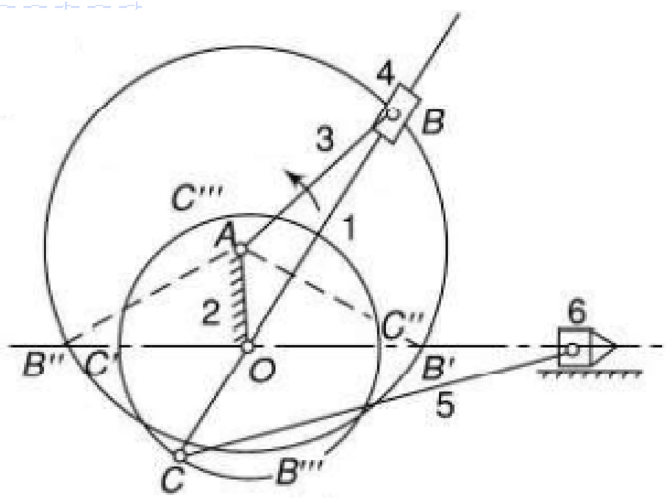
Kinematic Inversions

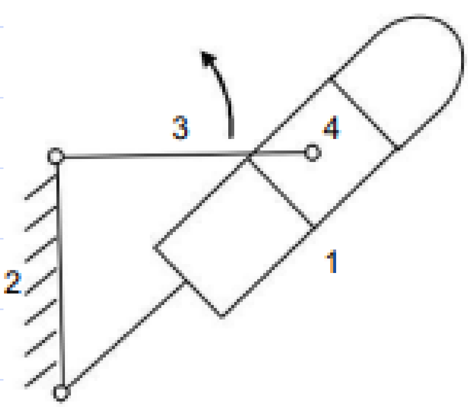
Inversions of Kinematic Chain with three revolute pairs and one sliding pair



Second Inversion





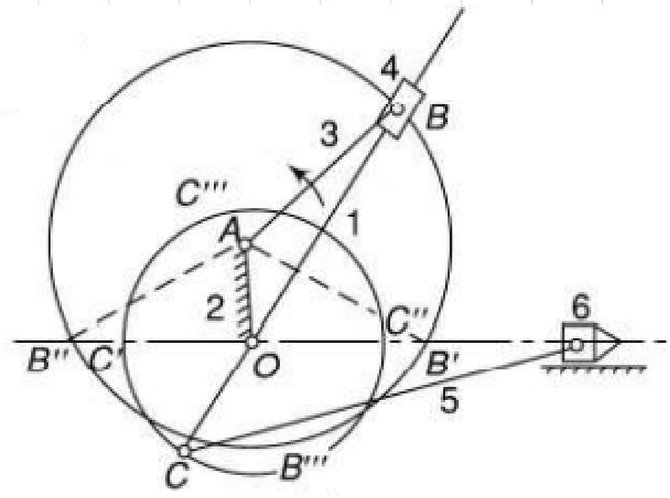


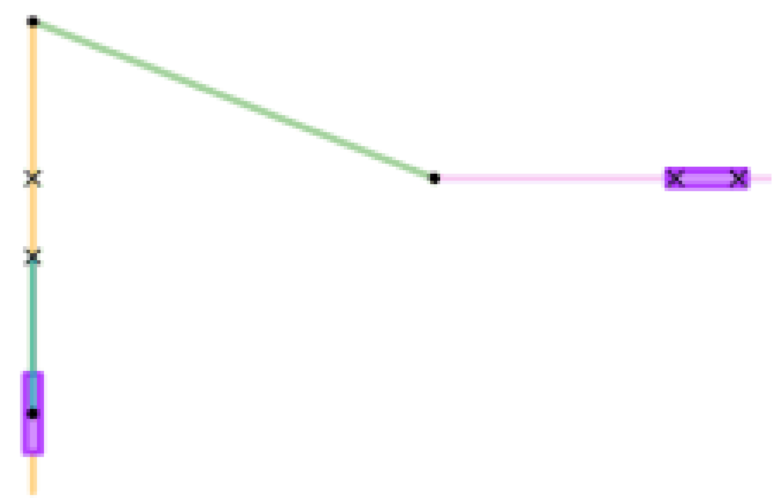
Whitworth Quick Return Motion Mechanism

Kinematic Inversions

Inversions of Kinematic Chain with three revolute pairs and one sliding pair

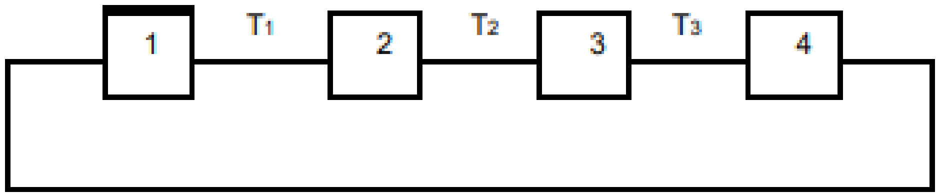
- The forward stroke starts when link 3 occupies position AB'. At that time, point C is at C'.
- The forward stroke ends when link 3 occupies position AB'' and point C occupies position C''.
- The return stroke takes place when link 3 moves from position AB'' to AB'.



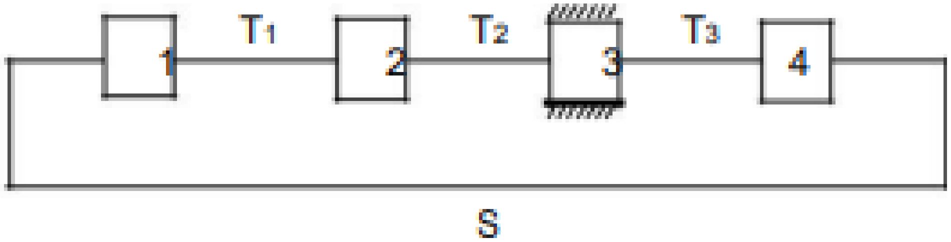


Kinematic Inversions

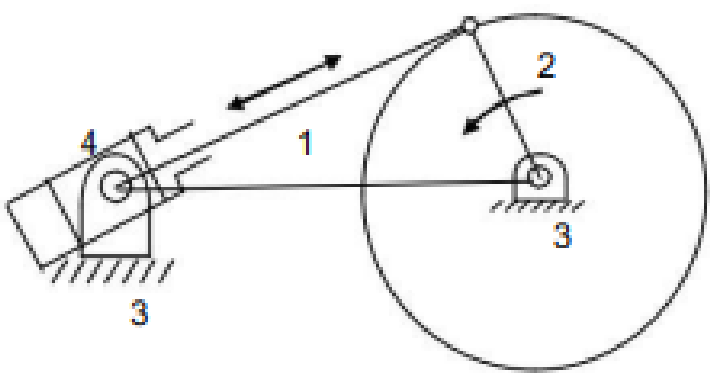
Inversions of Kinematic Chain with three revolute pairs and one sliding pair



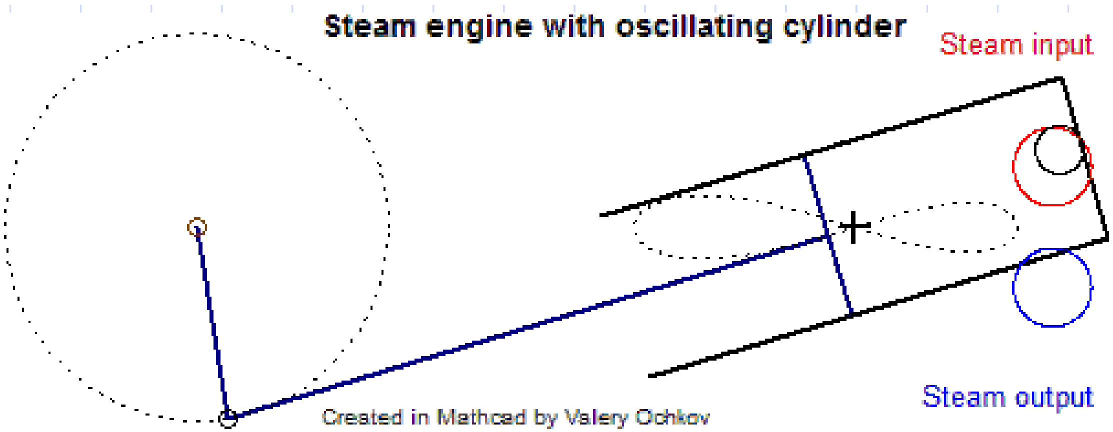
Third Inversion



- This inversion is obtained by fixing link 3.
- Link 1 works as a slider which slides in slotted or cylindrical link 4.
- Link 2 works as a crank.



(a) Oscillating Cylinder Engine



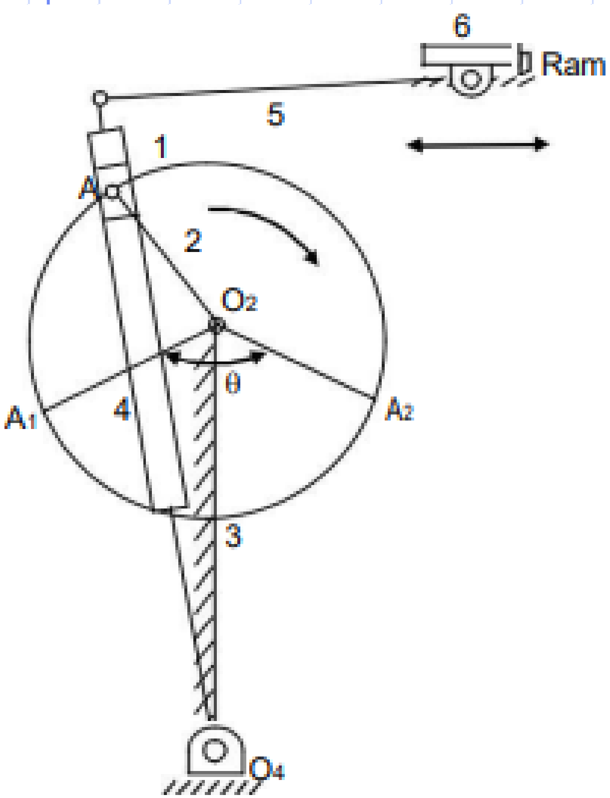
Kinematic Inversions

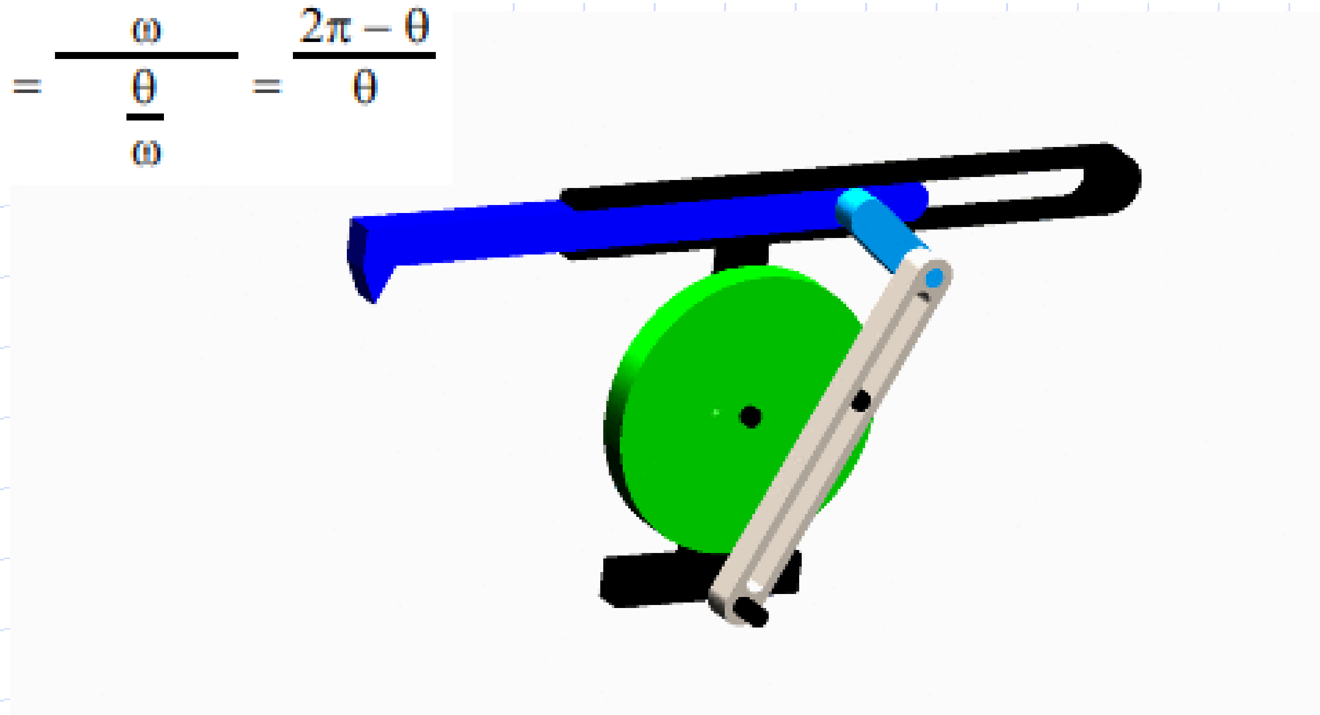
Third Inversion

- O2A1 and O2A2 are two positions of crank when link 4 will be tangential to the crank circle and corresponding to which ram will have extreme positions.
- When crank travels from position O2A1 and O2A2 forward stroke takes place.
- When crank moves [illegible] n stroke takes place.

$$\text{Quick Return Ratio} = \frac{\text{Time for forward stroke}}{\text{Time for return stroke}}$$

$$= \frac{\frac{(2\pi - \theta)}{\omega}}{\frac{\theta}{\omega}} = \frac{2\pi - \theta}{\theta}$$

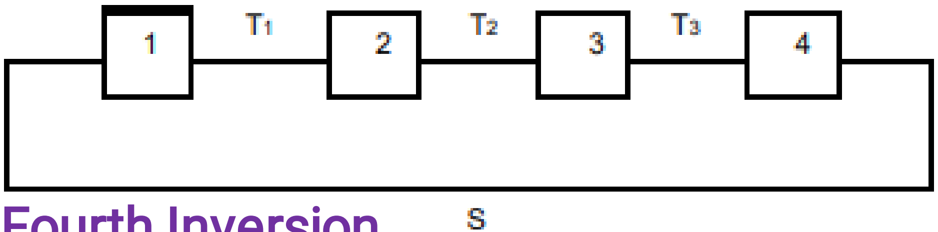




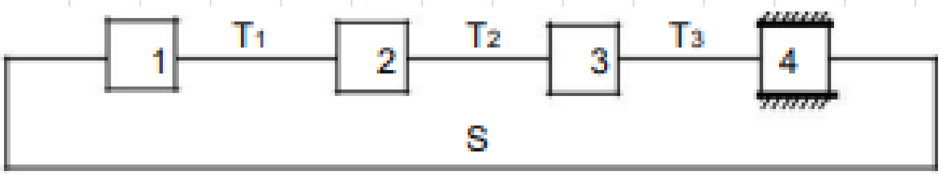
(b) Crank and Slotted Lever Mechanism

Kinematic Inversions

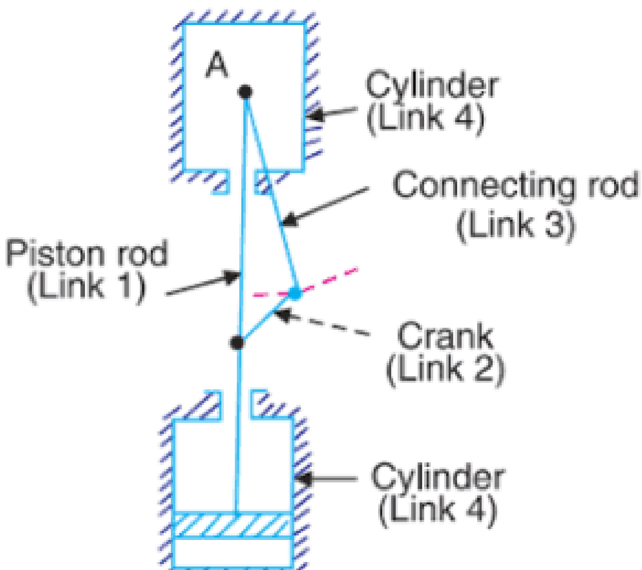
Inversions of Kinematic Chain with three revolute pairs and one sliding pair



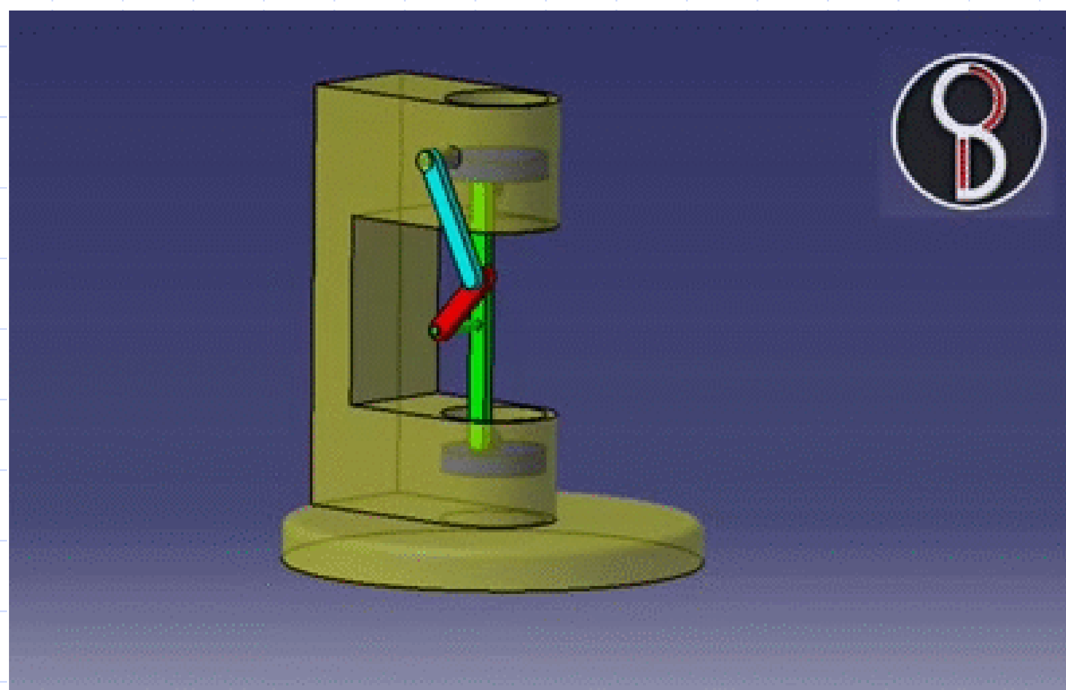
Fourth Inversion



- This inversion is obtained by fixing link 4.
- The pendulum pump and hand pump are examples of this inversion.

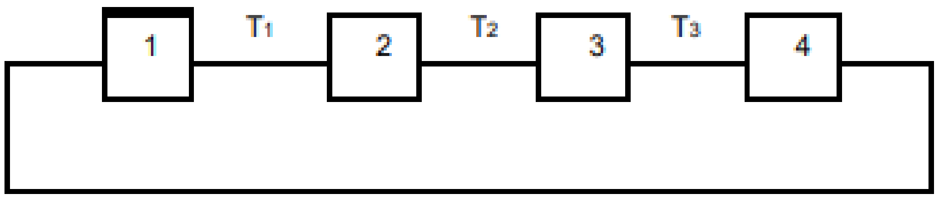


Pendulum

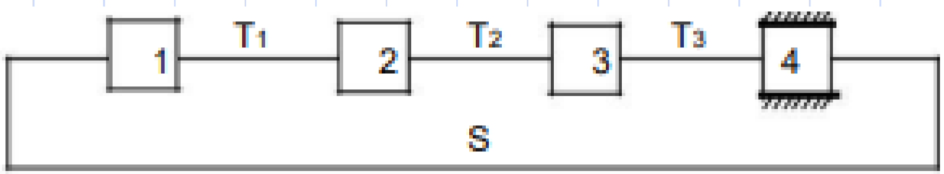


Kinematic Inversions

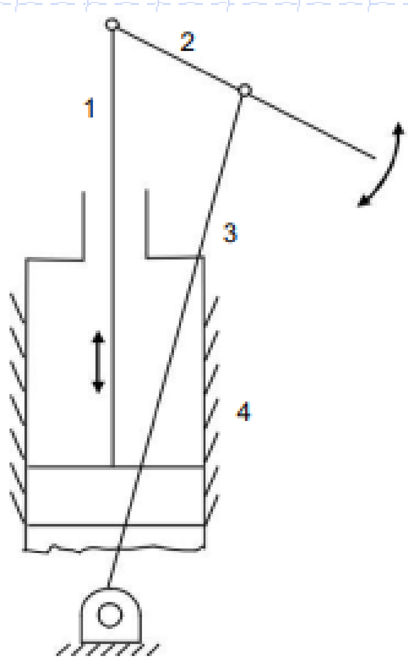
Inversions of Kinematic Chain with three revolute pairs and one sliding pair

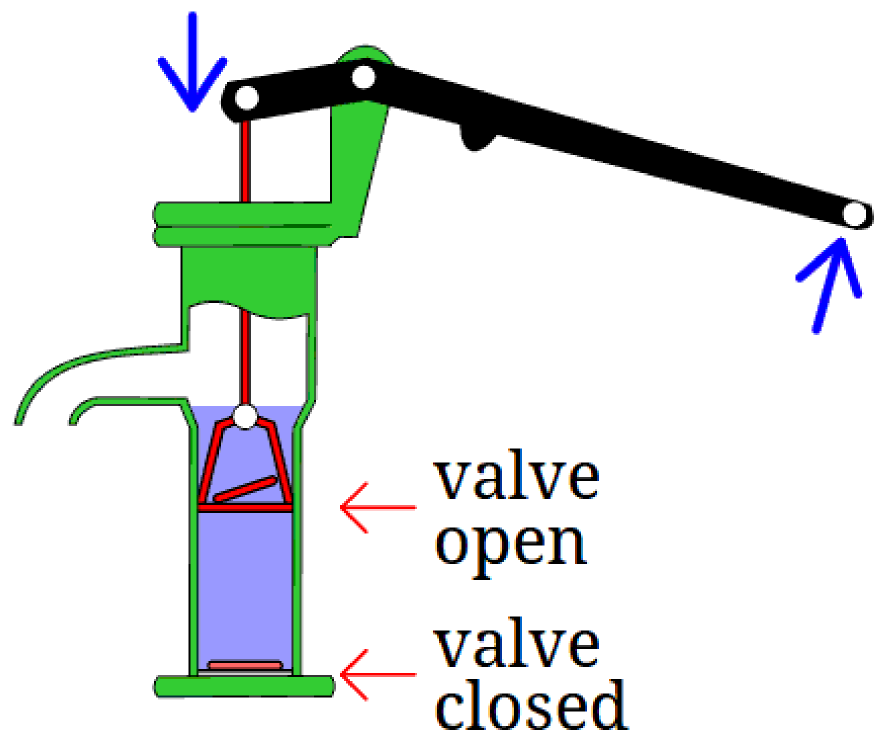


Fourth Inversion



Hand Pump





Kinematic Inversions

P1: Discuss in detail the kinematic inversions of Double Slider Crank Chain.

Mechanisms with Lower Pair

Paucellier Mechanism

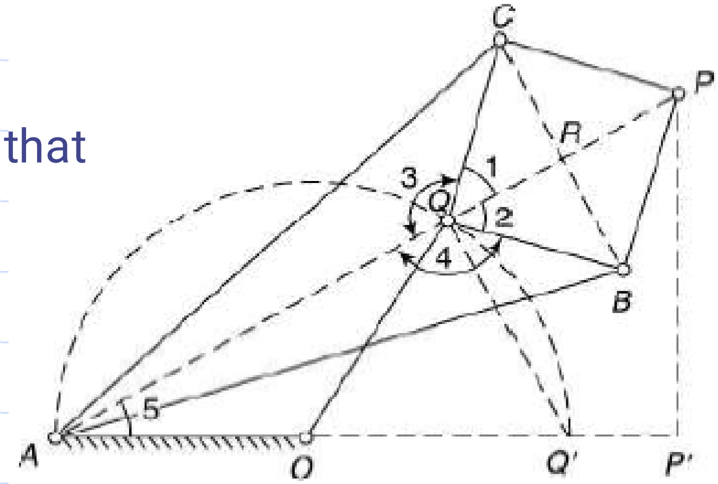
A Paucellier mechanism consists of eight links such that

$$OA = OQ; \qquad AB = AC$$

$$BP = PC = CQ = QB$$

OA: Fixed link

OQ: Rotating Link



As the link OQ moves around O, P moves in a straight line perpendicular to OA.
All the joints are pin-jointed.

Since $BPCQ$ is a rhombus,
QP always bisects the angle BQC,
i.e.,

$$\angle 1 = \angle 2 \qquad \text{(i)}$$

in all the positions
Also, in Δs AQC and AQB,
AQ is common,
$AC = AB$
$QC = QB$
∴ Δs are congruent in all positions.

$$\Rightarrow \quad \angle 3 = \angle 4 \qquad \text{(ii)}$$

Adding (i) and (ii),
$\angle 1 + \angle 3 = \angle 2 + \angle 4$
But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$

$$\therefore \quad \angle 1 + \angle 3 = \angle 2 + \angle 4 = 180^\circ$$

Or A,Q and P lie on a straight line.
Let $PP' \perp AO$

$$\Delta AQQ' \sim \Delta AP'P$$

$$\therefore \frac{AQ}{AP'} = \frac{AQ'}{AP}$$

or $AQ' . AP' = AQ . AP$

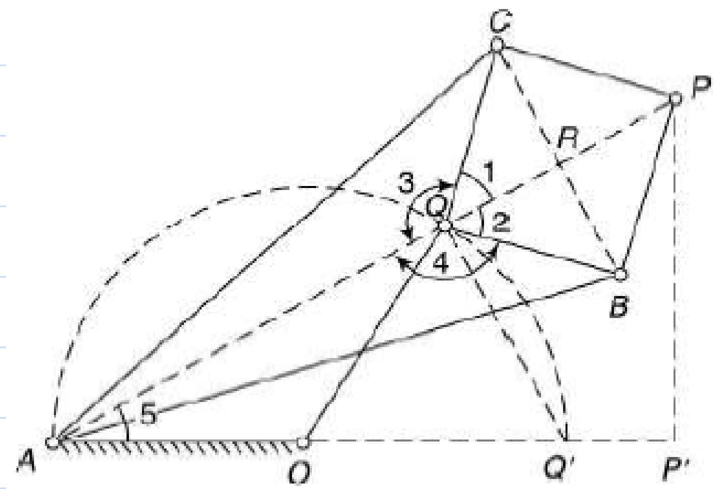
Paucellier Mechanism

or $AQ'.AP' = AQ.AP$

$$= (AR - RQ)(AR + RP)$$
$$= (AR - RQ)(AR + RQ)$$
$$= (AR)^2 - (RQ)^2$$
$$= [(AC)^2 - (CR)^2) - [(CQ)^2 - (CR)^2]$$

$$AP' = \frac{(AC)^2 - (CQ)^2}{AQ'}$$

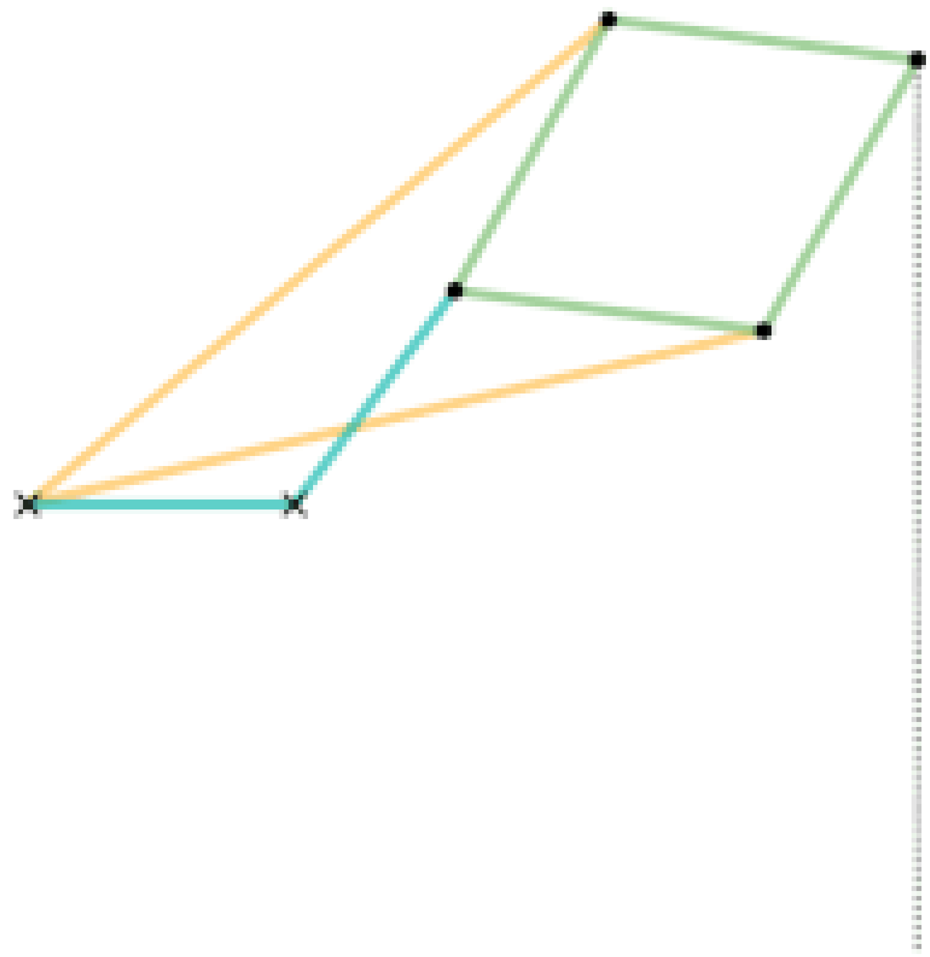
= constant, as AC, CQ and AQ' are always fixed



This means that the projection of *P* on *AO* produced is constant for all the configurations. or *P* moves in a straight line perpendicular to *AO*.

STRAIGHT-LINE MECHANISMS

Paucellier Mechanism



STRAIGHT-LINE MECHANISMS

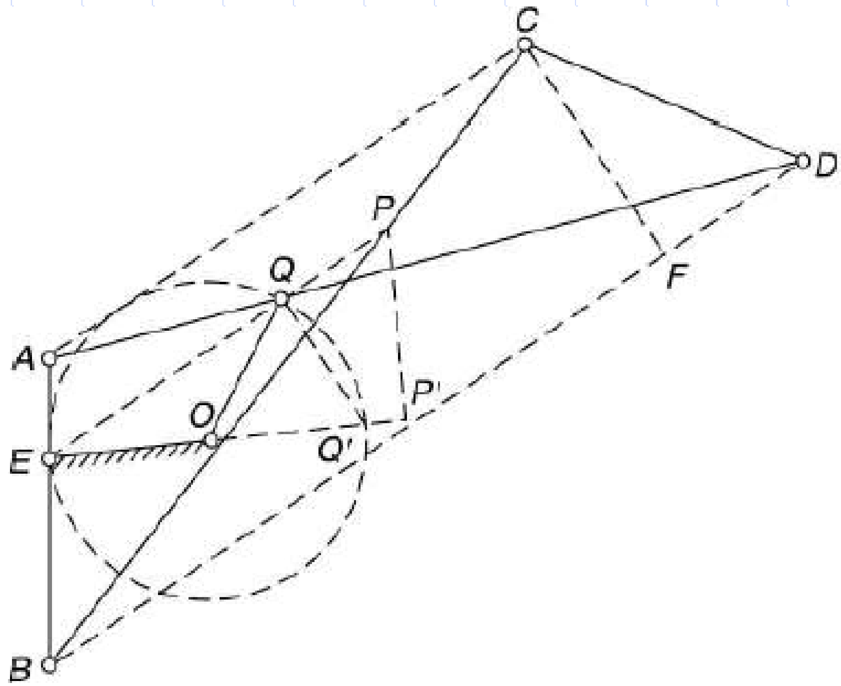
Hart Mechanism

- This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.

$AB = CD; \quad AD = BC \quad \text{and } OE = OQ$

OE is the fixed link and OQ, the rotating link.

- ABCD is a trapezium



$$\frac{AE}{AB} = \frac{AQ}{AD} = \frac{CP}{CB} \qquad \text{(i)}$$

- OQ rotates about O, P moves in a line perpendicular to EO produced.

$$\text{In } \Delta ABD \quad \frac{AE}{AB} = \frac{AQ}{AD} \qquad \text{(Given)}$$

$\therefore$ EQ is parallel to BD and thus parallel to AC.

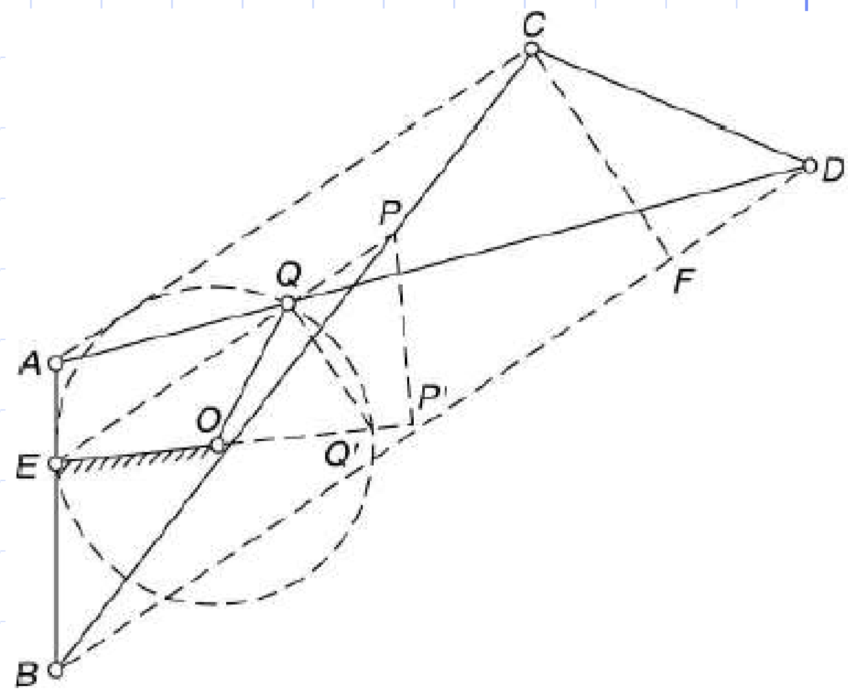
$$\text{In } \Delta ABC \quad \frac{AE}{AB} = \frac{CP}{CB} \qquad \text{(Given)}$$

$\therefore$ EP is parallel to AC and thus parallel to BD.

$\Rightarrow$ EQP is a straight line

STRAIGHT-LINE MECHANISMS

Hart Mechanism



Δs AEQ and ABD are similar ($\because$ $EQ \parallel BD$).

$$\therefore \frac{EQ}{BD} = \frac{AE}{AB} \text{ or } EQ = BD \times \frac{AE}{AB} \qquad \text{(ii)}$$

Δs BEP and BAC are similar ($\because$ $EP \parallel AC$).

$$\therefore \frac{EP}{AC} = \frac{BE}{BA} \text{ or } EP = AC \times \frac{BE}{AB} \qquad \text{(iii)}$$

Δs EQQ' and $EP'P$ are similar

$\because$ $\angle QEQ'$ or $\angle PEP'$ is common

and $\angle EQQ' = \angle QP'P = 90°$.

$$\therefore \frac{EQ}{EP'} = \frac{EQ'}{EP}$$

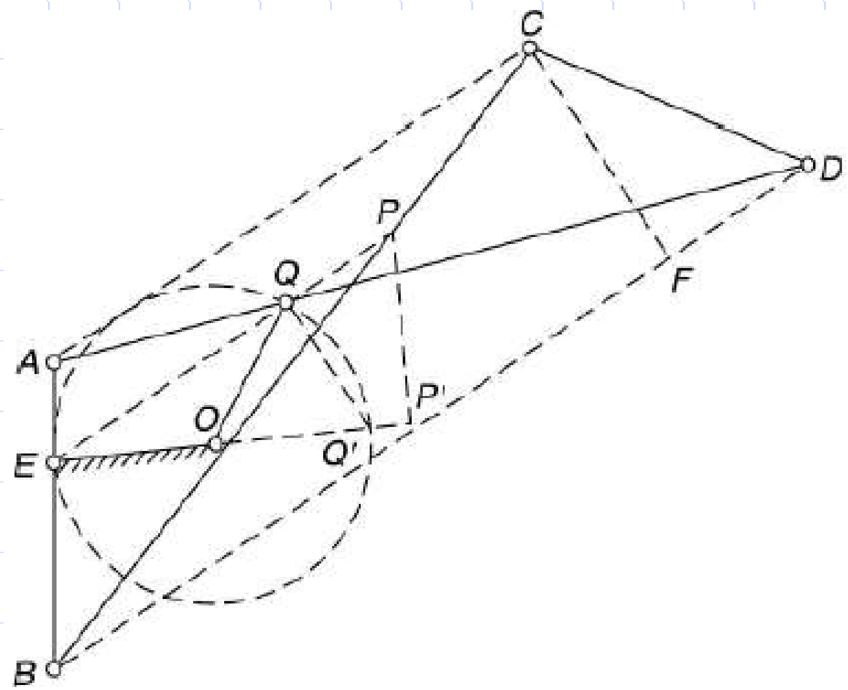
$$EP' = \frac{AE \times BE}{(EQ')(AB)^2}[(BD)(AC)] \qquad \text{[from (ii) and (iii)]}$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BF + FD)(BF - FD)\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BF)^2 - (FD)^2\right]$$

STRAIGHT-LINE MECHANISMS

Hart Mechanism



$$EP' = \frac{AE \times BE}{(EQ')(AB)^2}\left[\{(BC)^2 - (CF)^2\} - \{(CD)^2 - (CF)^2\}\right]$$

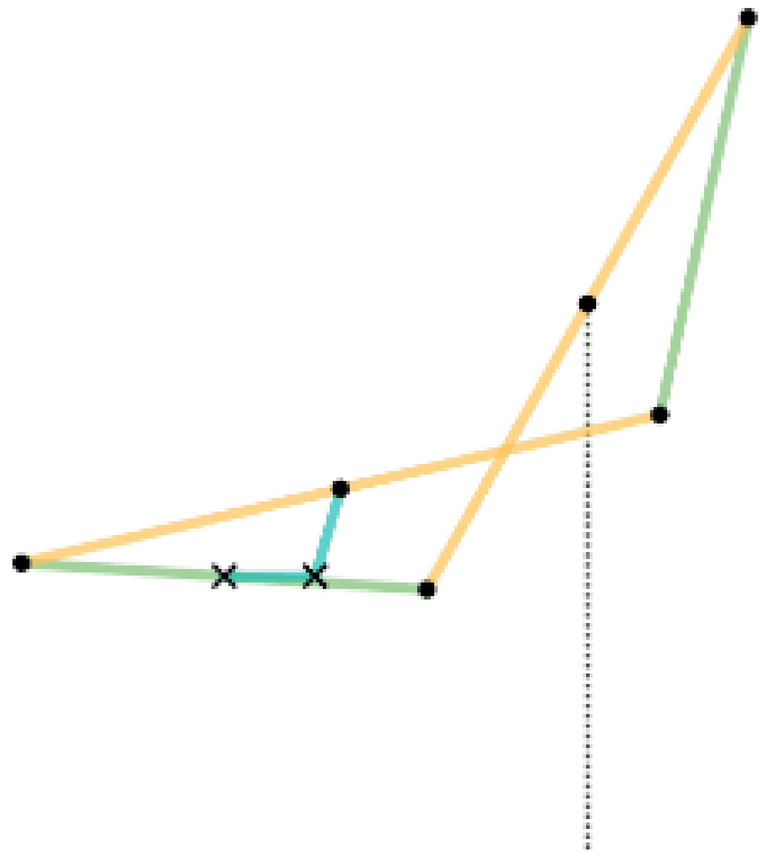
$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BC)^2 - (CD)^2\right]$$

= constant, as all the parameters are fixed.

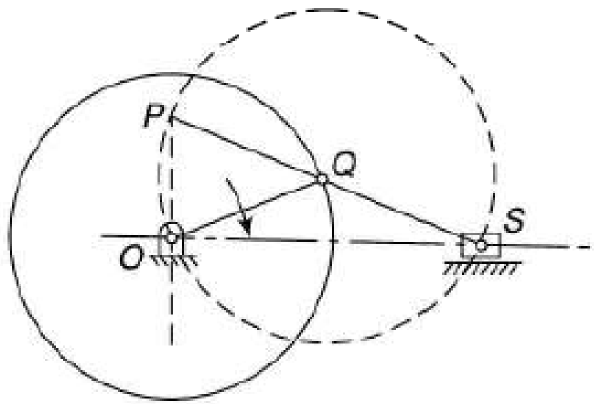
∴ The projection of P on EO produced is always the same point or P moves in a straight line perpendicular to EO.

STRAIGHT-LINE MECHANISMS

Hart Mechanism



Scott-Russel Mechanism



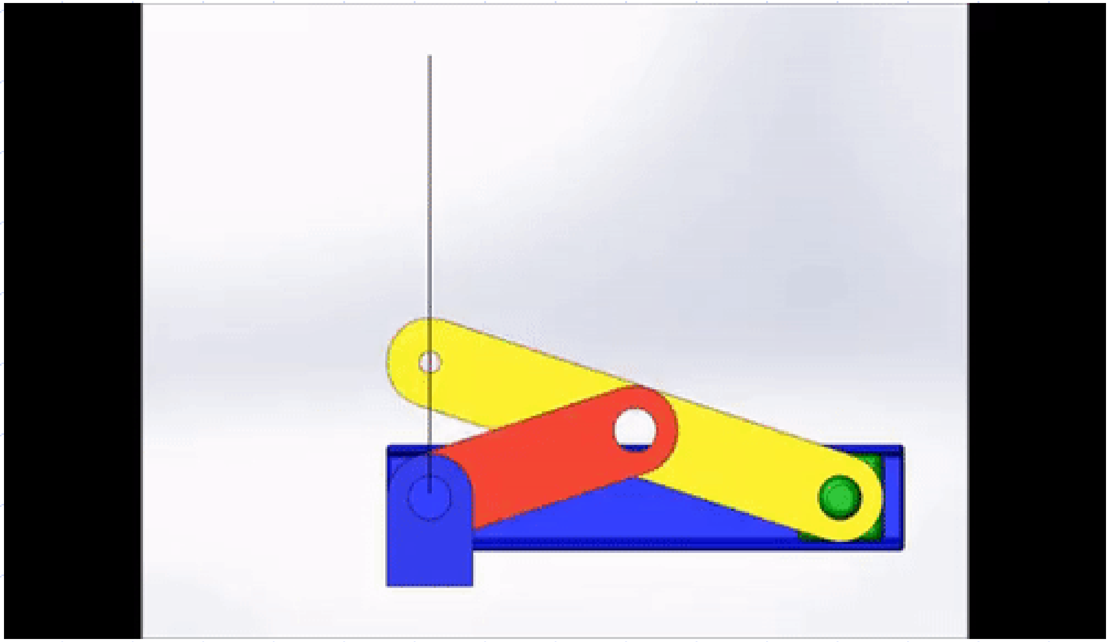
A Scott–Russel mechanism consists of three movable links; *OQ*, *PS* and slider *S* which moves along *OS*. *OQ* is the crank.

The links are connected in such a way that

$$QO = QP = QS$$

P moves in a straight line perpendicular to *OS* at *O* as the slider *S* moves along *OS*.

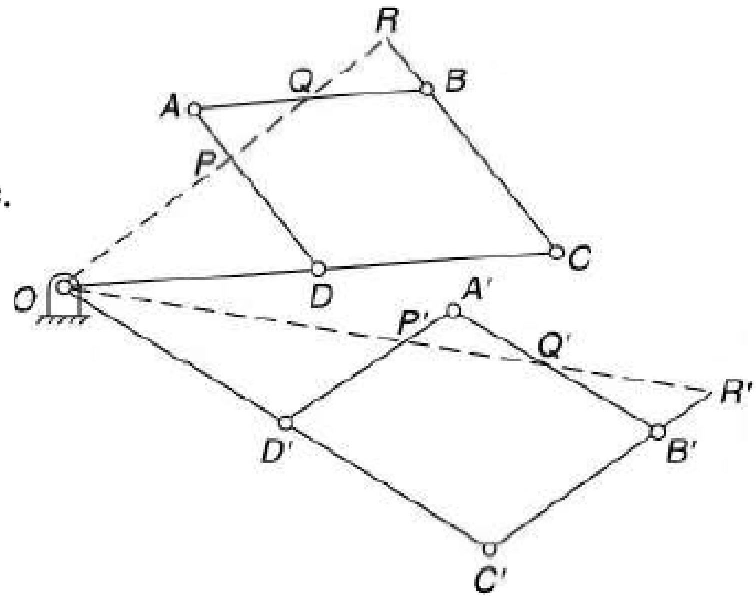
Scott-Russel Mechanism



P2: Discuss in detail three Approximate Straight Line Motion Mechanisms.

Pantograph

- A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point.
  - The four links of a pantograph are arranged in such a way that a parallelogram ADCD is formed.
  - AB = DC and BC = AD
  - O,P,Q and R lie on links CD, DA, AB and BC respectively such that OPQR is a straight line.
  - ABCD is the initial assumed position.
  - $A'B'C'D'$ is the new position.



In ΔODP and OCR,

O, P and R lie on a straight line and thus OP and OR coincide.

$\angle DOP = \angle COR$ (common angle)

$\angle ODP = \angle OCR$ ($\because DP \,||\, CR$)

Therefore, the Δs are similar and

$$\frac{OD}{OC} = \frac{OP}{OR} = \frac{DP}{CR}$$

Now, $A'B' = AB = DC = D'C'$

And $B'C' = BC = AD = A'D'$

Therefore, $A'B'C'D'$ is again a parallelogram.

In Δs $OD'P'$ and $OC'R'$,

$$\frac{OD'}{OC'} = \frac{OD}{OC} = \frac{DP}{CR}$$

$$= \frac{D'P'}{C'R'}$$

and,

$$\angle OD'P' = \angle OC'R' \qquad (D'P' \parallel C'B' \text{ as } A'B'C'D' \text{ is a } \parallel \text{gm})$$

Thus the triangles $OD'P'$ and $OC'R'$ are similar triangles.

$\therefore \qquad \angle D'OP' = \angle C'OR'$

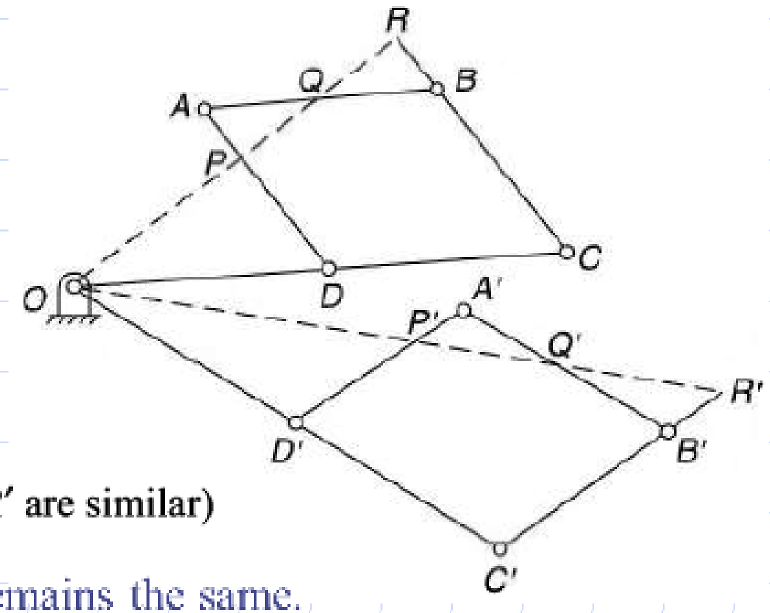
or $\quad O$, P' and R' lie on a straight line.

Now

$$\frac{OP}{OR} = \frac{OD}{OC}$$

$$= \frac{OD'}{OC'}$$

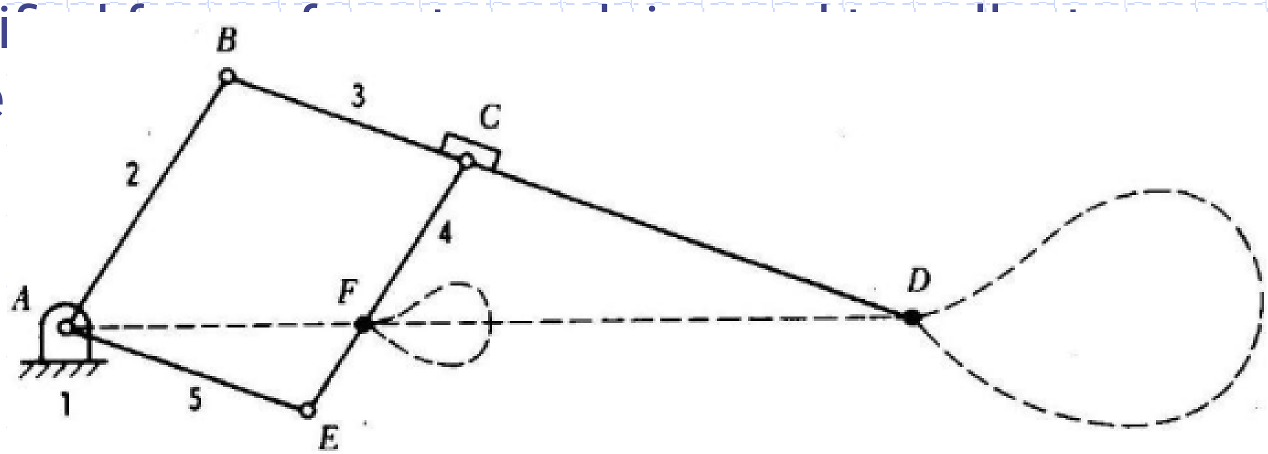
$$= \frac{OP'}{OR'} \qquad (\because \Delta s\ OD'P' \text{ and } OC'R' \text{ are similar})$$



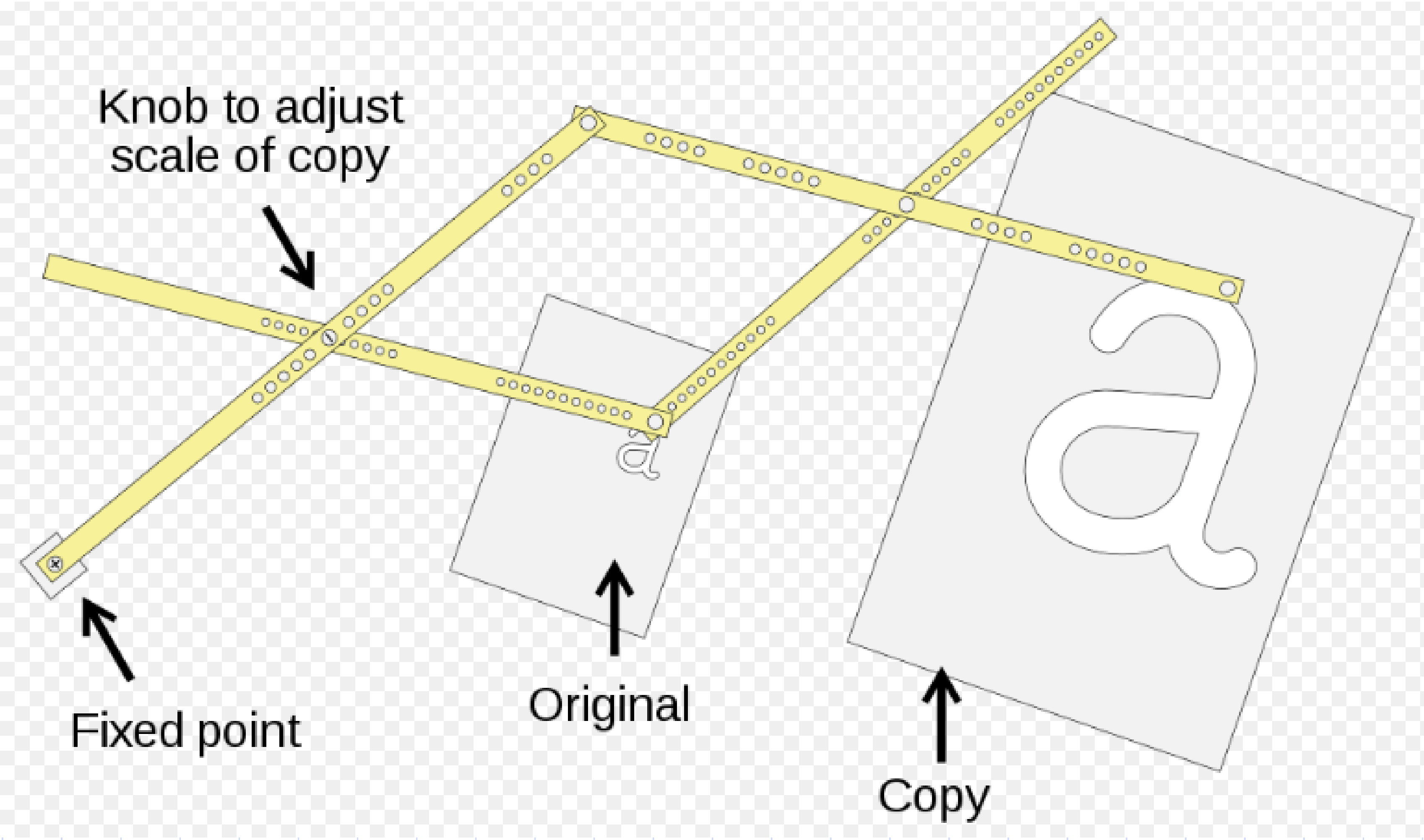
$\therefore$The ratio of distances of P and R from the fixed point remains the same.
This will be true for all the positions of the links.
Thus, P and R will trace exactly similar paths.

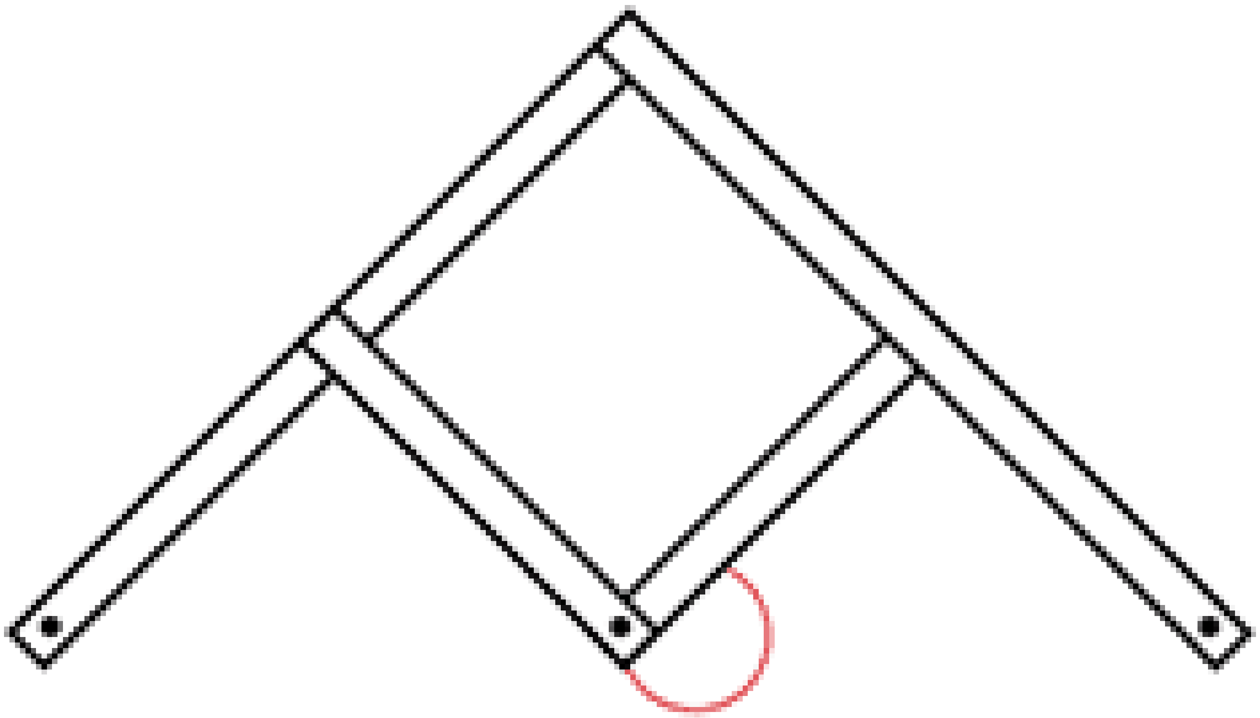
Uses of Pantograph

- A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point.
- A pantograph is mostly used for the reproduction of plane areas and figures such as maps, plans etc., on enlarged or reduced scales.
- It is, sometimes, used as an indicator rig in order to reproduce to a small scale the displacement of the crosshead and therefore of the piston of a reciprocating steam engine. It is also used to guide cutting tools.
- A modi at the top of an e



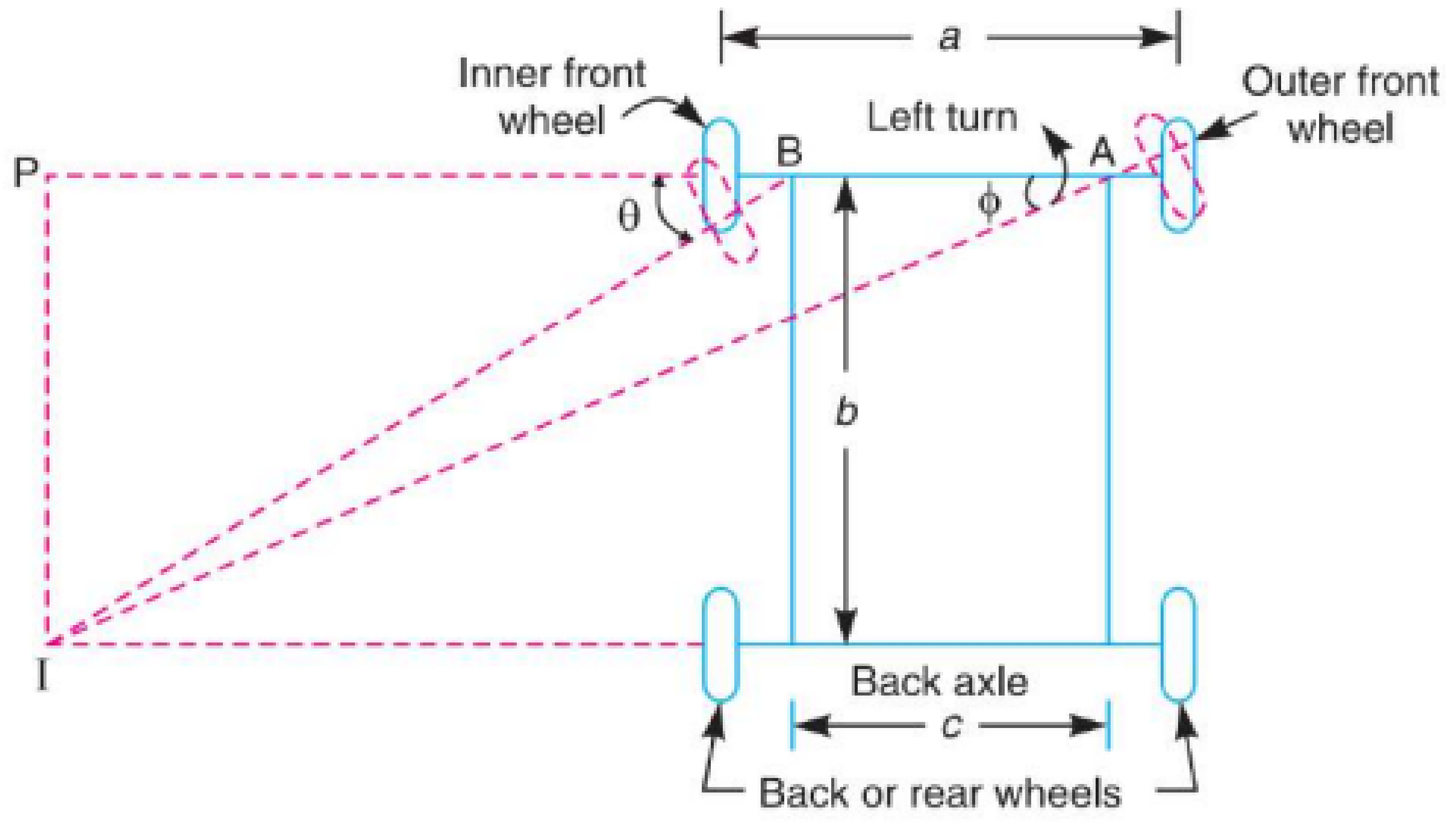
Uses of Pantograph





STEERING GEAR MECHANISM

- The steering gear mechanism is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path.
- Usually the two back wheels have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels.



STEERING GEAR MECHANISM

- Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre.
- The axis of the inner wheel makes a larger turning angle θ than the angle ϕ subtended by the axis of outer wheel.

Let a = Wheel track,

b = Wheel base, and

c = Distance between the pivots A and B of the front axle.

Now from triangle IBP,

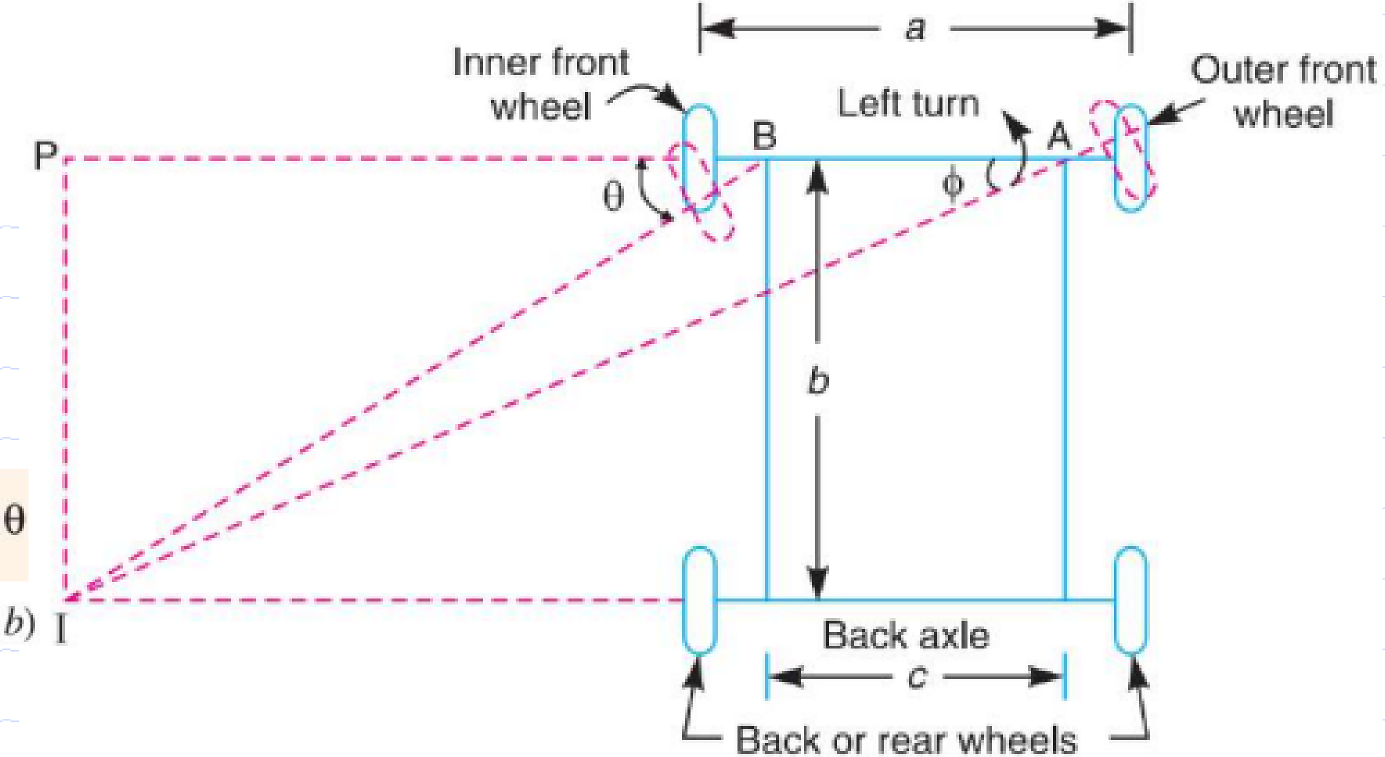
$$\cot\theta = \frac{BP}{IP}$$

and from triangle IAP,

$$\cot\phi = \frac{AP}{IP} = \frac{AB + BP}{IP}$$

$$= \frac{AB}{IP} + \frac{BP}{IP} = \frac{c}{b} + \cot\theta$$

...($\because$ $IP = b$)



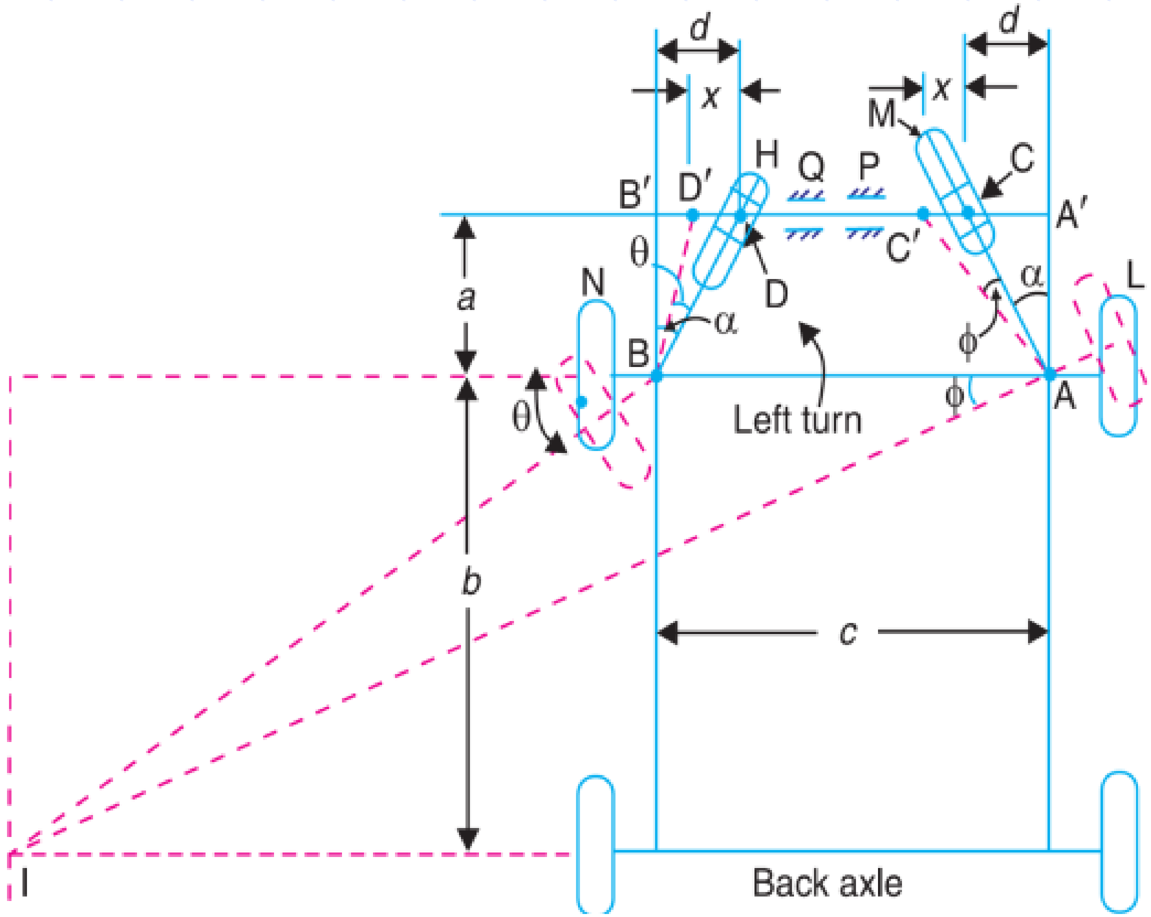
$\therefore \cot\phi - \cot\theta = c/b$

- This is the fundamental equation for correct steering. If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.

DAVIS STEERING GEAR MECHANISM

- It is an exact steering gear mechanism. The slotted links AM and BH are attached to the front wheel axle, which turn on pivots A and B respectively.
- The rod CD is constrained to move in the direction of its length, by the sliding members at P and Q. These constraints are connected to the slotted link AM and BH by a sliding and a turning pair at each end.

a = Vertical distance between AB and CD,
b = Wheel base,
$2d$ = Difference between AB and CD ,
c = Distance between the pivots A and B of the front axle.
x = Distance moved by C to C' = CC' = DD', and
α = Angle of inclination of the links AC and BD, to the vertical.



DAVIS STEERING GEAR MECHANISM

From triangle $AA'C'$,

$$\tan(\alpha + \phi) = \frac{A'C'}{AA'} = \frac{d + x}{a}$$

From triangle $AA'C$,

$$\tan \alpha = \frac{A'C}{AA'} = \frac{d}{a}$$

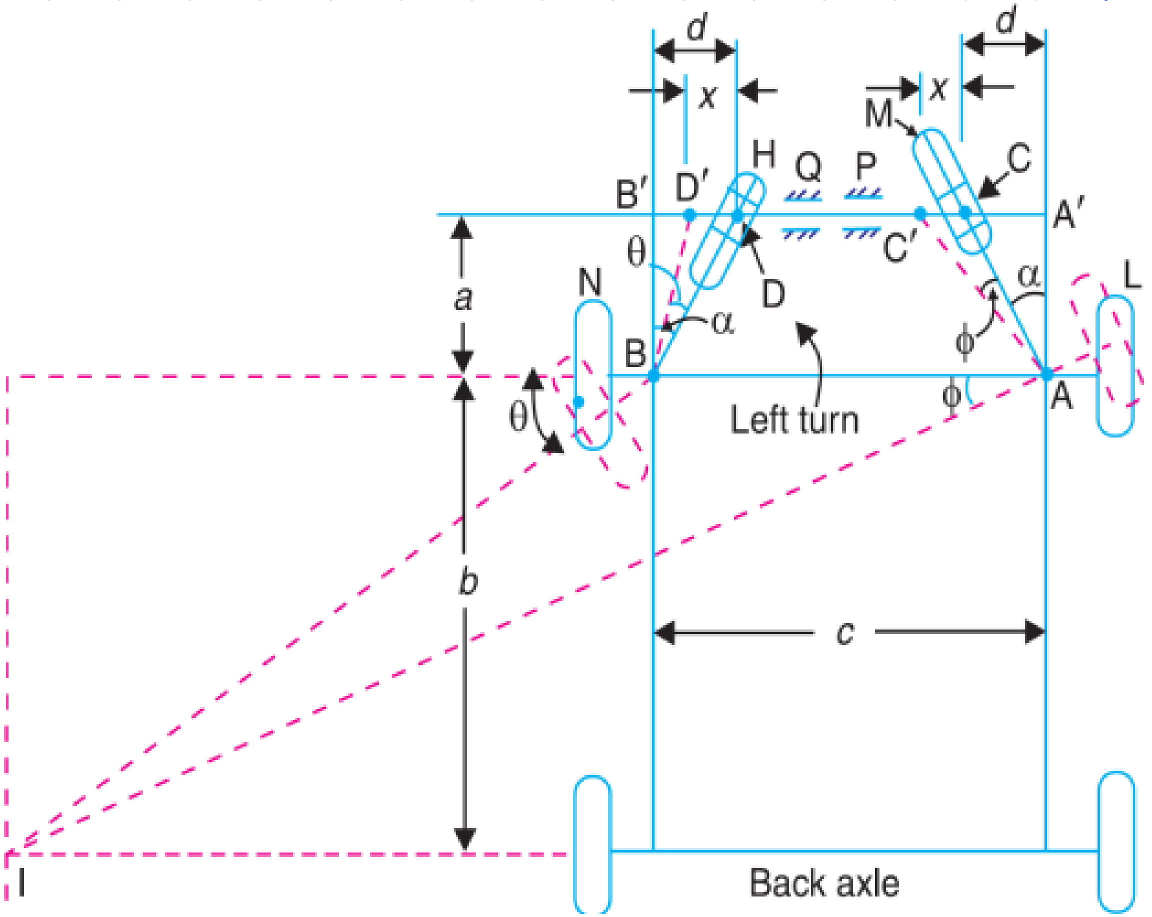
From triangle $BB'D'$,

$$\tan(\alpha - \theta) = \frac{B'D'}{BB'} = \frac{d - x}{a}$$

We know that

$$\tan(\alpha + \phi) = \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha . \tan \phi}$$

$$\frac{d + x}{a} = \frac{d/a + \tan \phi}{1 - d/a \times \tan \phi} = \frac{d + a \tan \phi}{a - d \tan \phi}$$



DAVIS STEERING GEAR MECHANISM

$$\frac{d+x}{a} = \frac{d/a + \tan\phi}{1 - d/a \times \tan\phi} = \frac{d + a\tan\phi}{a - d\tan\phi}$$

$$\Rightarrow \quad \tan\phi = \frac{a.x}{a^2 + d^2 + d.x}$$

Similarly, from

$$\tan(\alpha - \theta) = \frac{d - x}{a},$$ we get

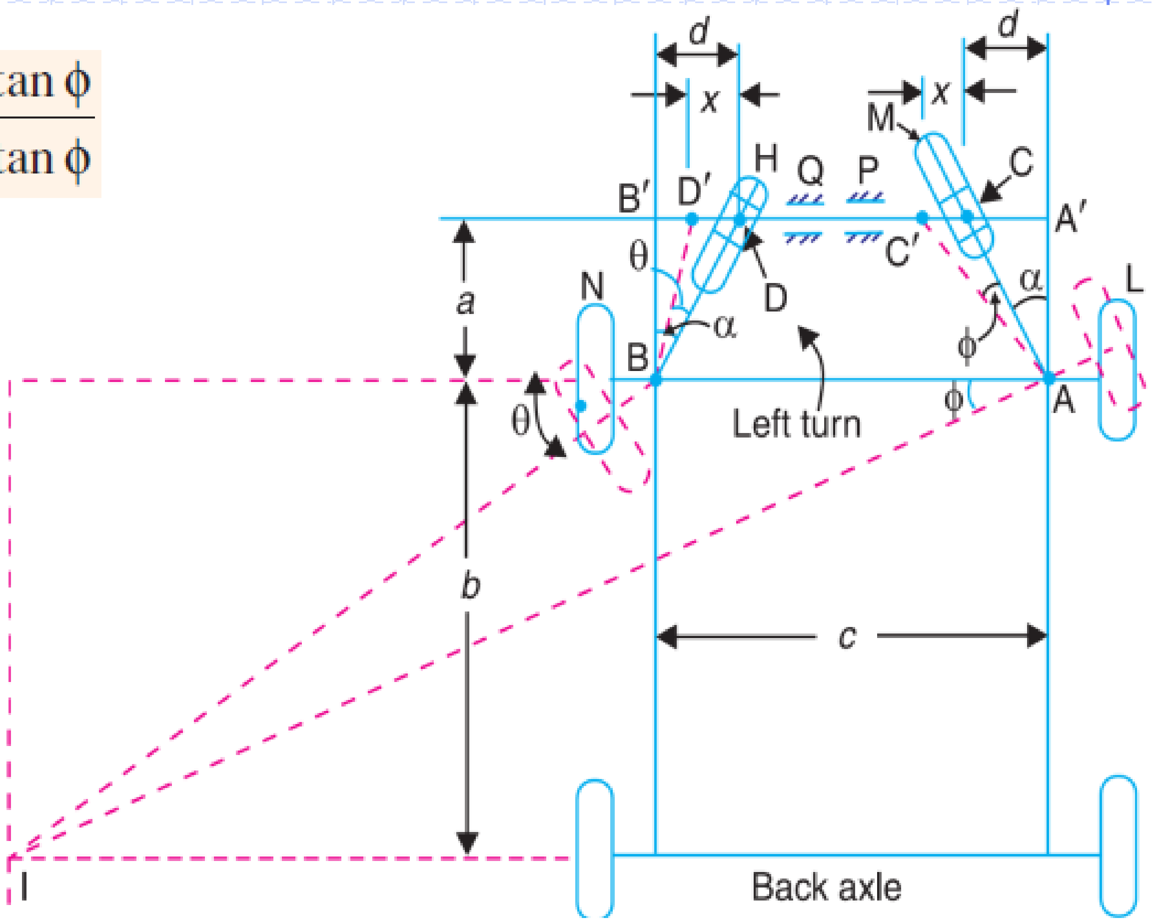
$$\tan\theta = \frac{ax}{a^2 + d^2 - d.x}$$

We know that for correct steering,

$$\cot\phi - \cot\theta = \frac{c}{b}$$ or

$$\frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b}$$ or $$\frac{a^2 + d^2 + d.x}{a.x} - \frac{a^2 + d^2 - d.x}{a.x} = \frac{c}{b}$$

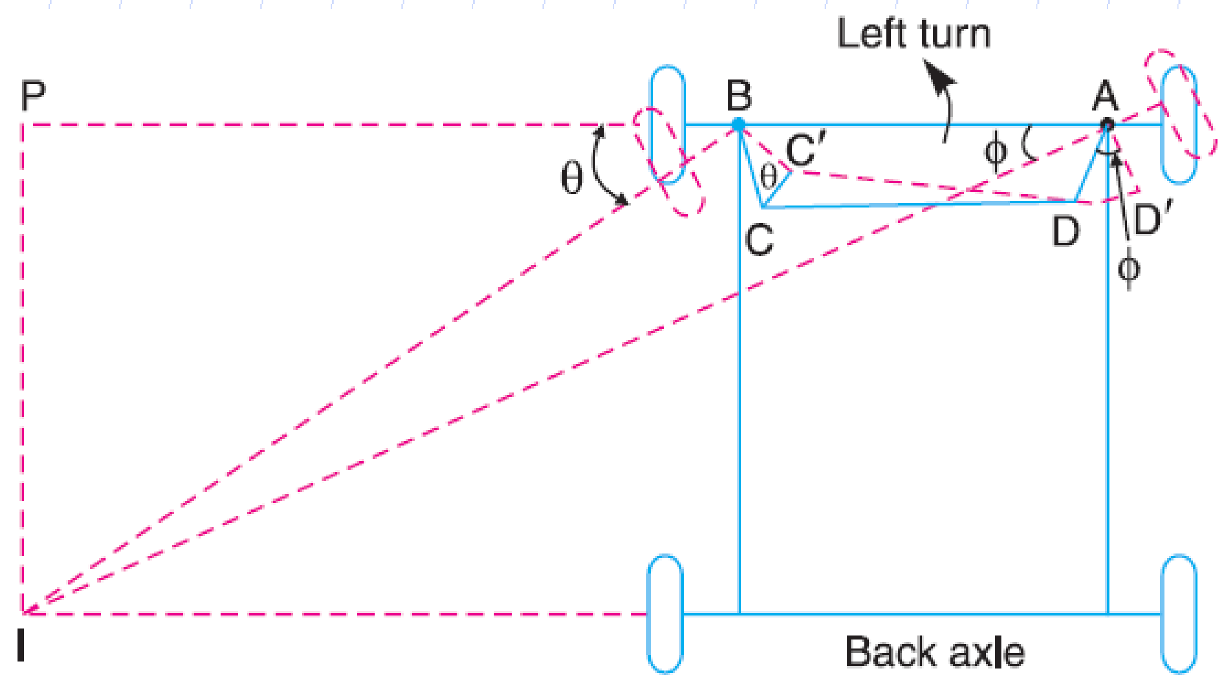
$$\Rightarrow \quad \tan\alpha = \frac{c}{2b}$$



- The usual value of $\frac{c}{2b}$ is between 0.4 to 0.5 and that of α from 11 to 14 degrees.

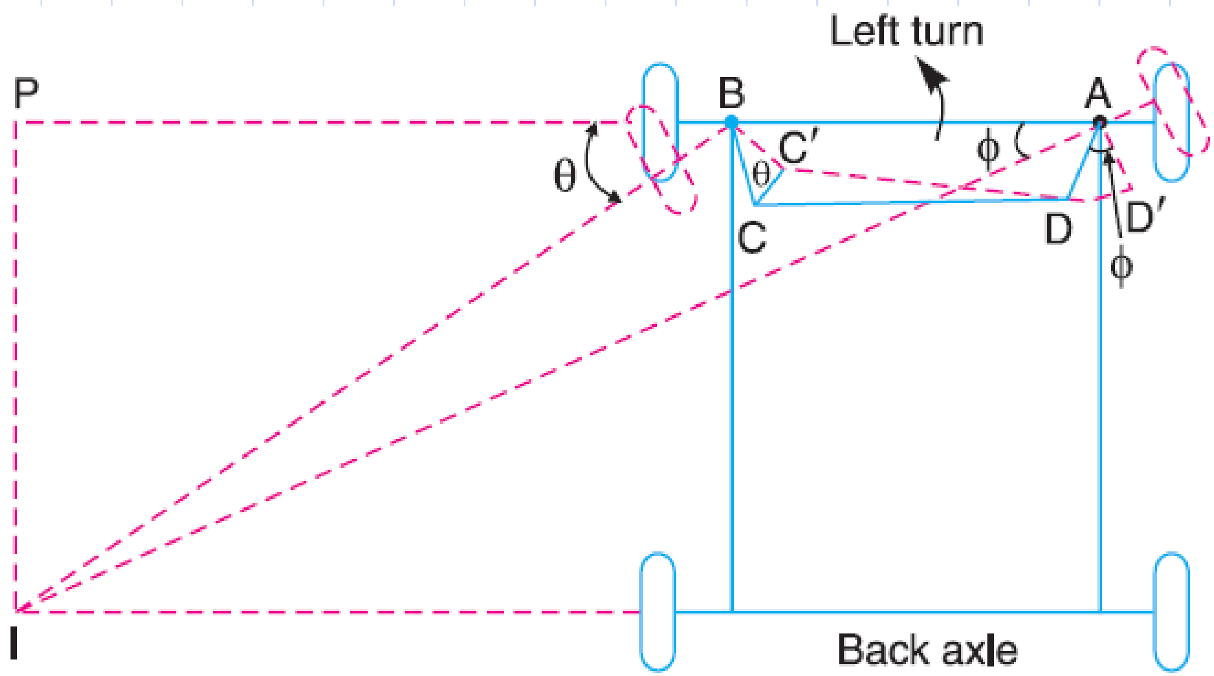
ACKERMANN STEERING GEAR MECHANISM

- The Ackerman steering gear **mechanism is much simpler than Davis gear.** The difference between the Ackerman and Davis steering gears are:
  - The whole mechanism of the **Ackerman steering gear is on back of the front wheels; whereas in Davis steering gear, it is in front of the wheels.**
  - The Ackerman **steering gear consists of turning pairs,** whereas Davis steering gear consists of sliding members.
- The **shorter links BC and AD are of equal length** and are connected by hinge joints with front wheel axles.
- The **longer links AB and CD are of unequal length.**

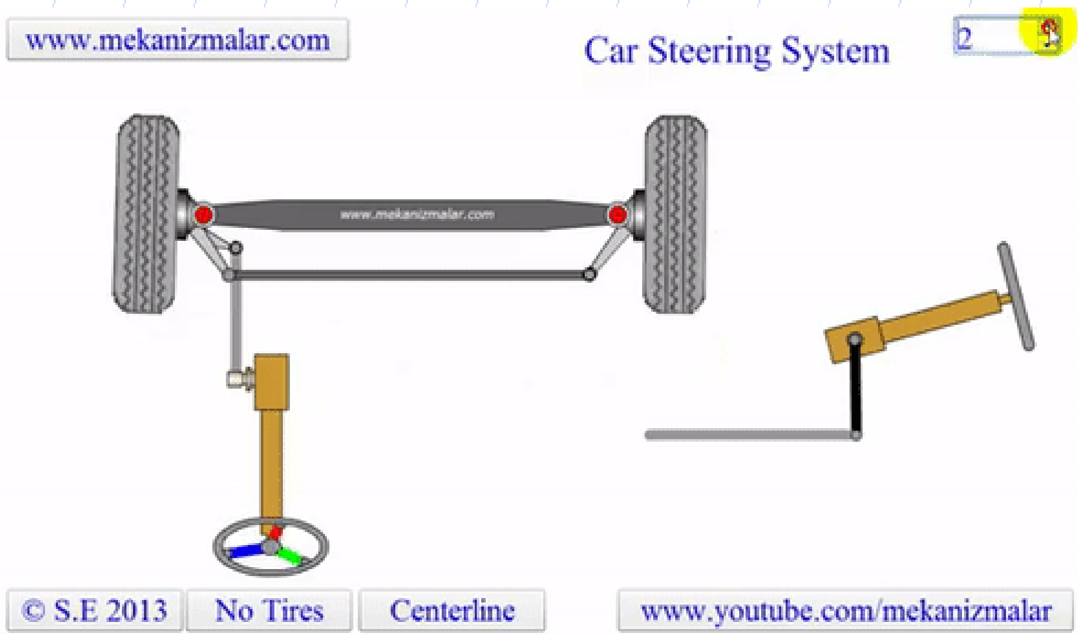


ACKERMANN STEERING GEAR MECHANISM

- The following are the only three positions for correct steering.
  - When the vehicle moves along a straight path, the longer links A B and CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle, as shown by firm lines in Fig. 2.12.
  - When the vehicle is steering to the left, the position of the gear is shown by dotted lines in Fig. In this position, the lines of the front wheel axle intersect on the back wheel axle at I, for correct steering.
  - When the vehicle is steering to the right, the similar position may be obtained.




ACKERMANN STEERING
4-BAR STEERING
ACKERMANN STEERING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Car Steering System
2
www.mekanizmalar.com
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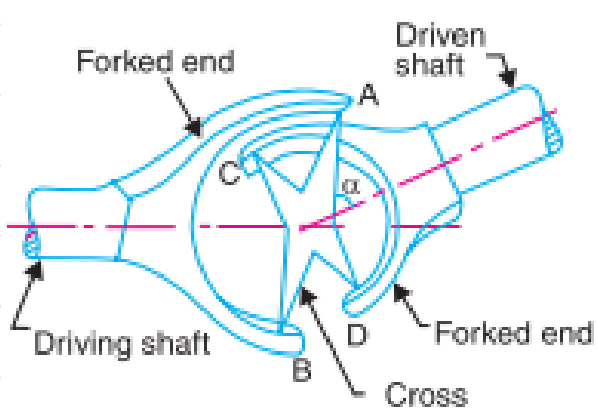
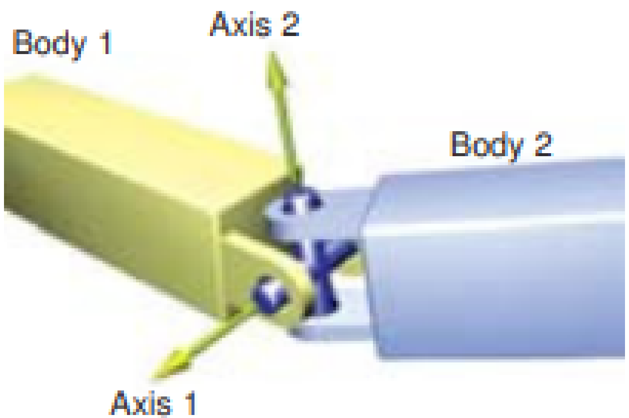
Hook's joint

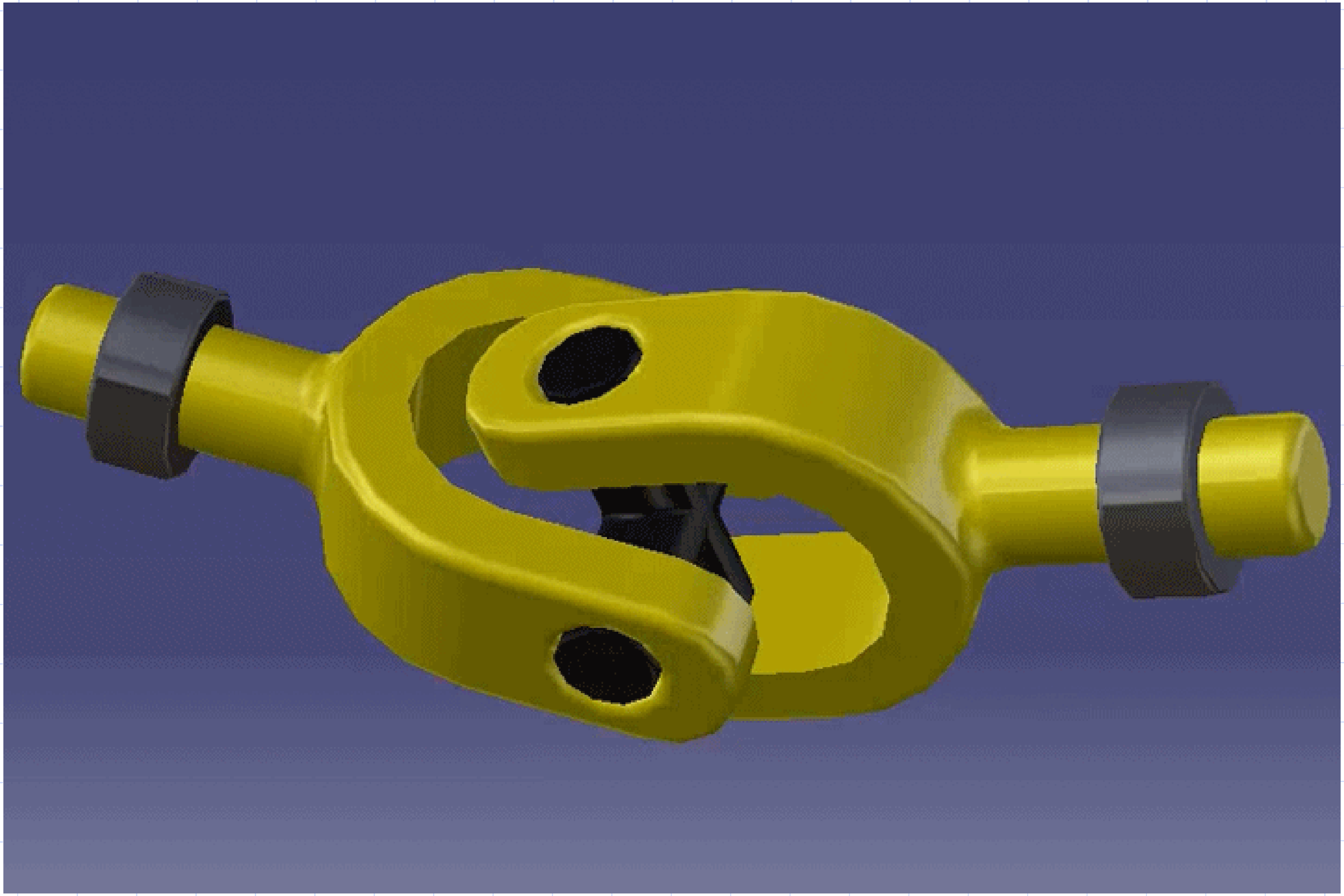
- A Hooke's joint is used to connect two non parallel and intersecting shafts.
- It is also used for shafts with angular misalignment.
- The driving shaft rotates at a uniform angular speed whereas the driven shaft rotates at a continuously varying angular speed.



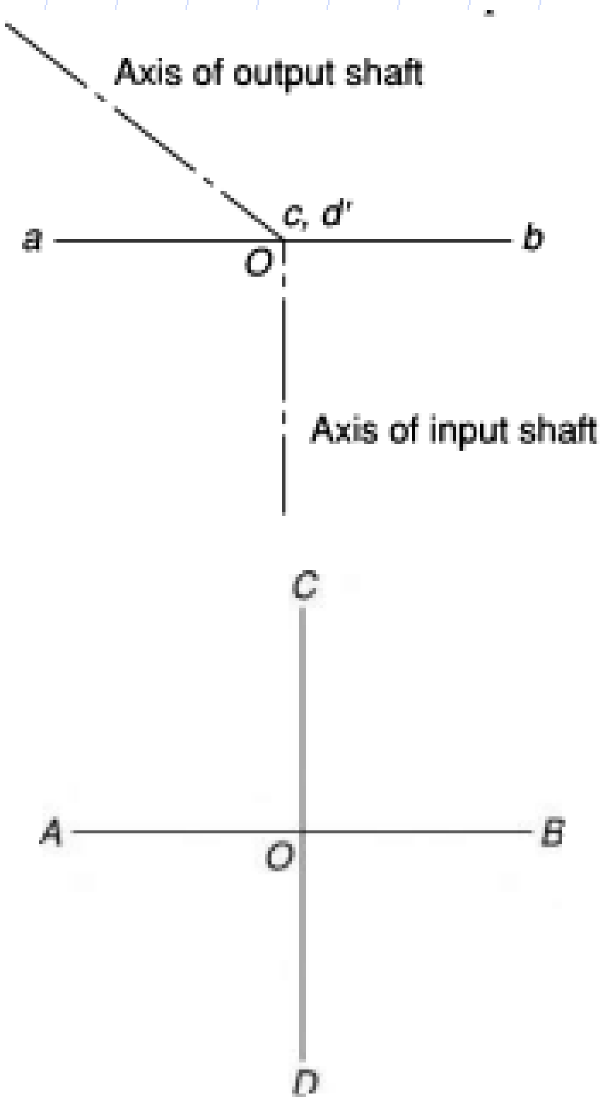


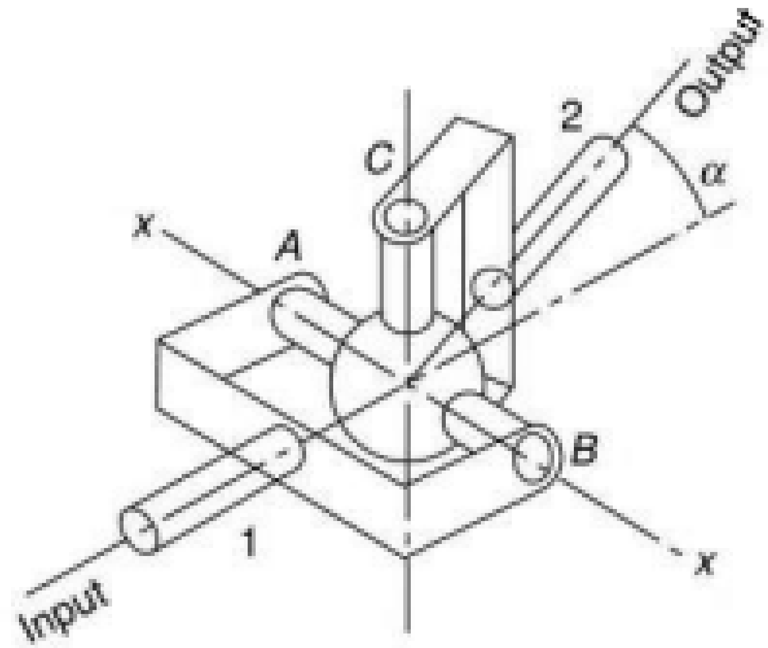
Applications:

- ✓ Used in automobiles where it is used to transmit power from the gear box of the engine to the rear axle.
- ✓ Used for transmission of power to different spindles of multiple drilling machine.
- ✓ Used as a knee joint in milling machines.



Hook's joint


Axis of output shaft
c, d'
a
b
O
Axis of input shaft
C
A
B
O
D


Output
2
C
α
x
A
B
1
x
Input

Hook's joint

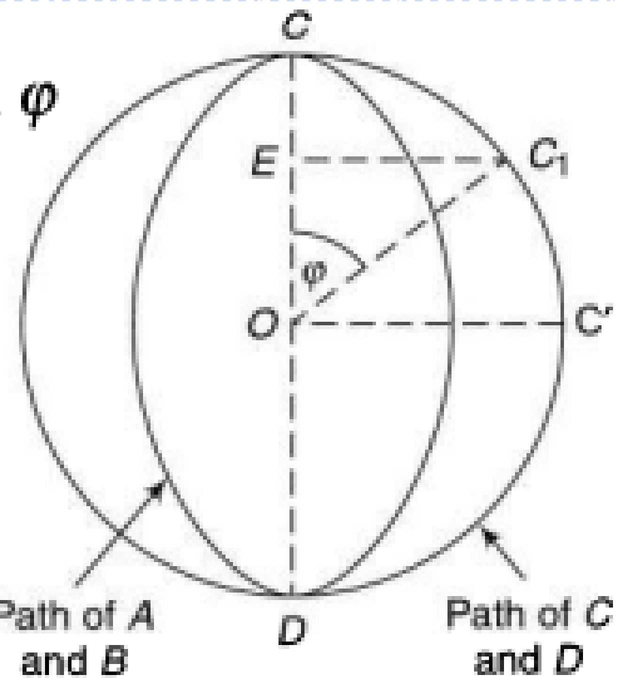
$$\frac{\tan\varphi}{\tan\theta} = \frac{EC_1' / EO}{EC_1 / EO} = \frac{EC_1'}{EC_1}$$

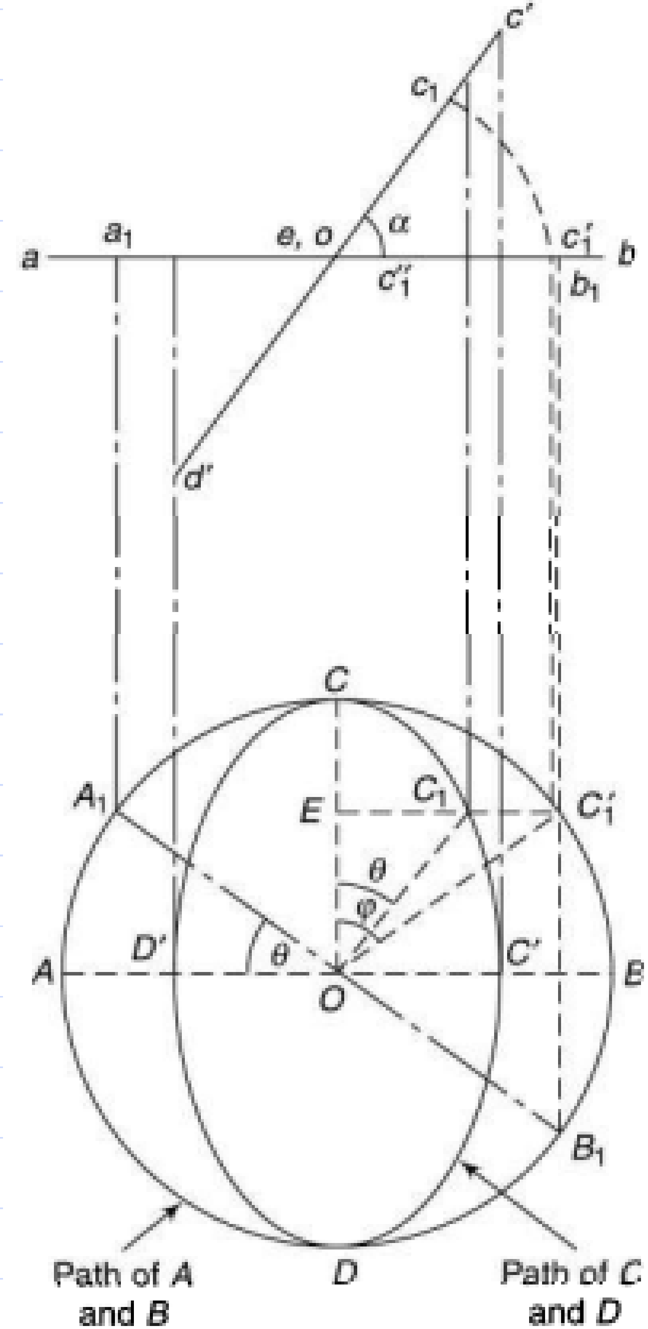
$$= \frac{ec_1'}{ec_1''} \quad \text{(from top view)}$$

$$= \frac{ec_1}{ec_1''} = \frac{1}{ec_1''/ec_1} = \frac{1}{\cos a}$$

$\Longrightarrow$

$$\tan\theta = \cos\alpha \tan\varphi$$



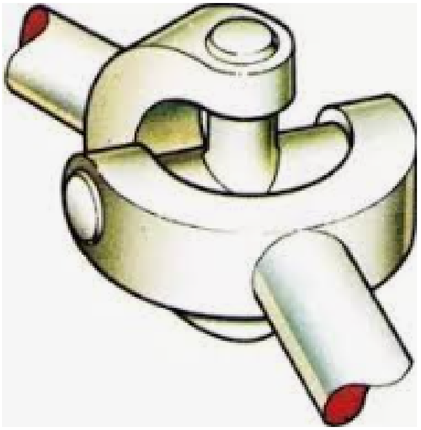


Hook's joint

Angular Velocity Ratio

ω_1 : Angular velocity of driving shaft
ω_2 : Angular velocity of driven shaft

$$\omega_1 = \frac{d\theta}{dt} \qquad \omega_2 = \frac{d\varphi}{dt}$$



We have

$$\omega_2 = \frac{d\varphi}{dt}$$

Differentiating the above equation w.r.t. time t

$$\sec^2\theta \frac{d\theta}{dt} = \cos\alpha \sec^2\varphi \frac{d\varphi}{dt}$$ or

$$\frac{\omega_2}{\omega_1} = \frac{1}{\cos^2\theta \cos\alpha (1 + \tan^2\varphi)} = \frac{1}{\cos^2\theta \cos\alpha \left(1 + \frac{\tan^2\theta}{\cos^2\alpha}\right)} = \frac{\cos\alpha}{\cos^2\theta(1 - \sin^2\alpha) + \sin^2\theta}$$

$$= \frac{\cos\alpha}{\cos^2\theta - \cos^2\theta \sin^2\alpha + \sin^2\theta} = \frac{\cos\alpha}{1 - \sin^2\alpha \cos^2\theta}$$

Hook's joint

Angular Velocity Ratio

1) For equal velocities of the driving and driven shafts, $\frac{\omega_2}{\omega_1} = 1$

But $\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}$

$$\therefore \quad \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta} = 1$$

or

$$\cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha} = \frac{1}{1+\cos\alpha} = \frac{\cos^2\theta\left(\frac{\sin^2\theta}{\cos^2\theta}+1\right)}{1+\cos\alpha}$$

or

$$\frac{\sin^2\theta}{\cos^2\theta}+1 = 1+\cos\alpha$$

or

$$\tan\theta = \pm\sqrt{\cos\alpha}$$

- Therefore velocities of the driven and driving shafts are equal once in all the four quadrants for particular values of θ if α is constant

Hook's joint

Angular Velocity Ratio

2) For minimum velocity ratio, $\frac{\omega_2}{\omega_1}$

$$\because \frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\theta}$$

$(1 - \sin^2\alpha\cos^2\theta)$ needs to be maximum

This happens when $\cos^2\theta$ is minimum

or $\theta = 90°$ or $270°$

Then $\frac{\omega_2}{\omega_1} = \cos\alpha$

3) For maximum velocity ratio, $\frac{\omega_2}{\omega_1}$

$$\because \frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\theta}$$

$(1 - \sin^2\alpha\cos^2\theta)$ needs to be minimum

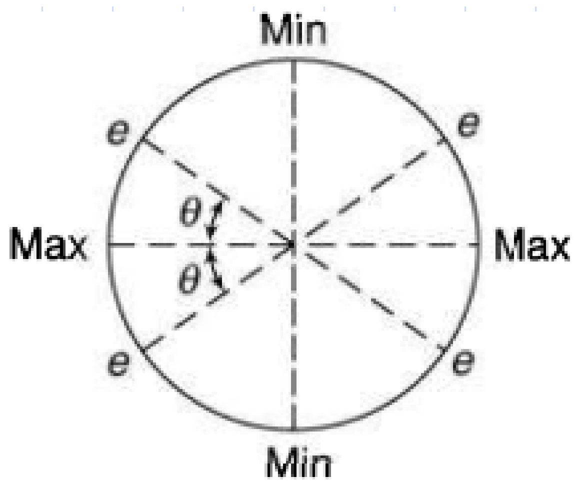
This happens when $\cos^2\theta$ is maximum

or $\theta = 0°$ or $180°$

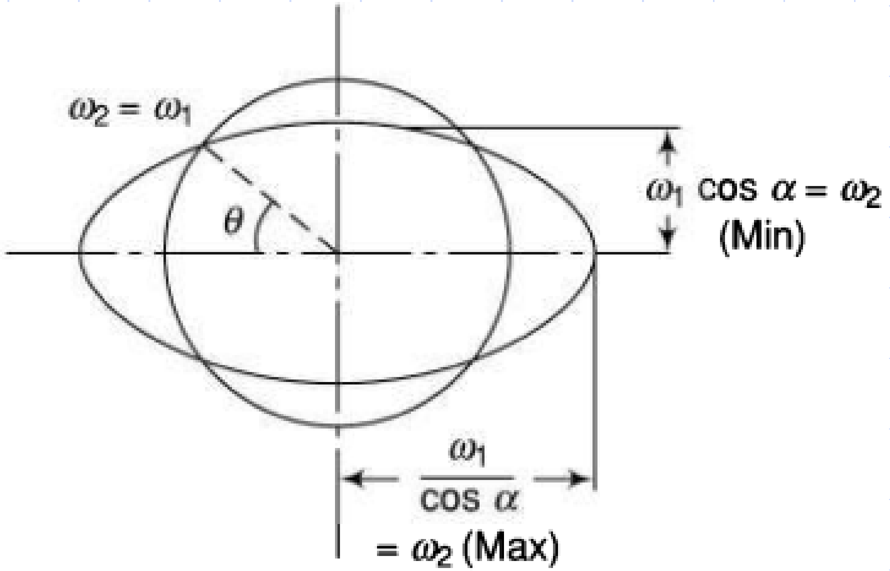
Then $\frac{\omega_2}{\omega_1} = \frac{1}{\cos\alpha}$

Hook's joint

Angular Velocity Ratio



Variation in the speed of driven shaft



Polar velocity Diagram

$$\text{Maximum variation} = \frac{\omega_{2\,\max} - \omega_{2\,\min}}{\omega_{\text{mean}}} = \frac{\omega_1 / \cos\alpha - \omega_1 \cos\alpha}{\omega_1}$$

$$= \tan\alpha \sin\alpha$$

$$\approx \alpha^2 \quad (\text{for small } \alpha)$$

Hook's joint

Angular Acceleration

ω_1: constant

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\theta}$$

For finding the acceleration, differentiating the above equation w.r.t.

$$\frac{d\omega_2}{dt} = \omega_1 \frac{d}{dt}\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}\right)$$

$$\text{acceleration} = \omega_1 . \frac{d\theta}{dt}\frac{d}{d\theta}\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}\right)$$

$$= \omega_1^2 \cos\alpha \frac{d}{d\theta}(1-\sin^2\alpha\cos^2\theta)^{-1}$$

$$= \frac{-\omega_1^2\cos\alpha\sin^2\alpha\sin 2\theta}{(1-\sin^2\alpha\cos^2\theta)^2}$$

Hook's joint

Angular Acceleration

Acceleration is minimum or maximum wh $\dfrac{d\,(acc)}{d\theta} = 0$

Which gives

$$\cos 2\theta \approx \frac{2\sin^2\alpha}{2-\sin^2\alpha}$$

Double Hook's joint

A universal joint, consisting of two Hooke's joints with an intermediate shaft, that eliminates variations in angular displacement and velocity between the driving and driven shafts

- This speed of the driving and driven shaft is constant
- This joint gives a velocity ratio equal to unity, if
  1. The axes of the driving and driven shafts are in the same plane, and
  2. The driving and driven shafts make equal angles with the intermediate shaft
  3. The two forks at the ends of intermediate shaft lie in the same plane.

$$\tan\theta = \cos\alpha \tan\gamma$$

$$\tan\varphi = \cos\alpha \tan\gamma$$

$$\therefore \theta = \varphi$$